EPFL

Steel structures, selected chapters

Fatigue: variable amplitude loading

(ref. TGC 10 section 13.5)

Part 4

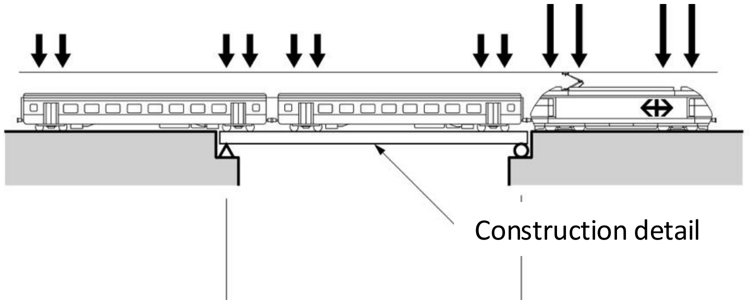
RESSLab
Resilient Steel Structures Laboratory

CONTENTS

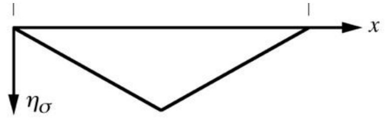
- **Variable amplitude loading**
- Simplification of a VA spectrum, cumulative damage law
- Variable amplitude fatigue verification formats
- Road traffic in Switzerland
- Simplified method, damage equivalence factor λ - determining length (and shape of influence line)

Variable amplitude loading

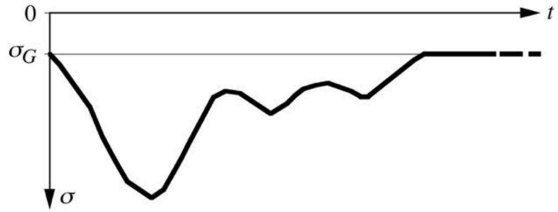
Loading model and static system



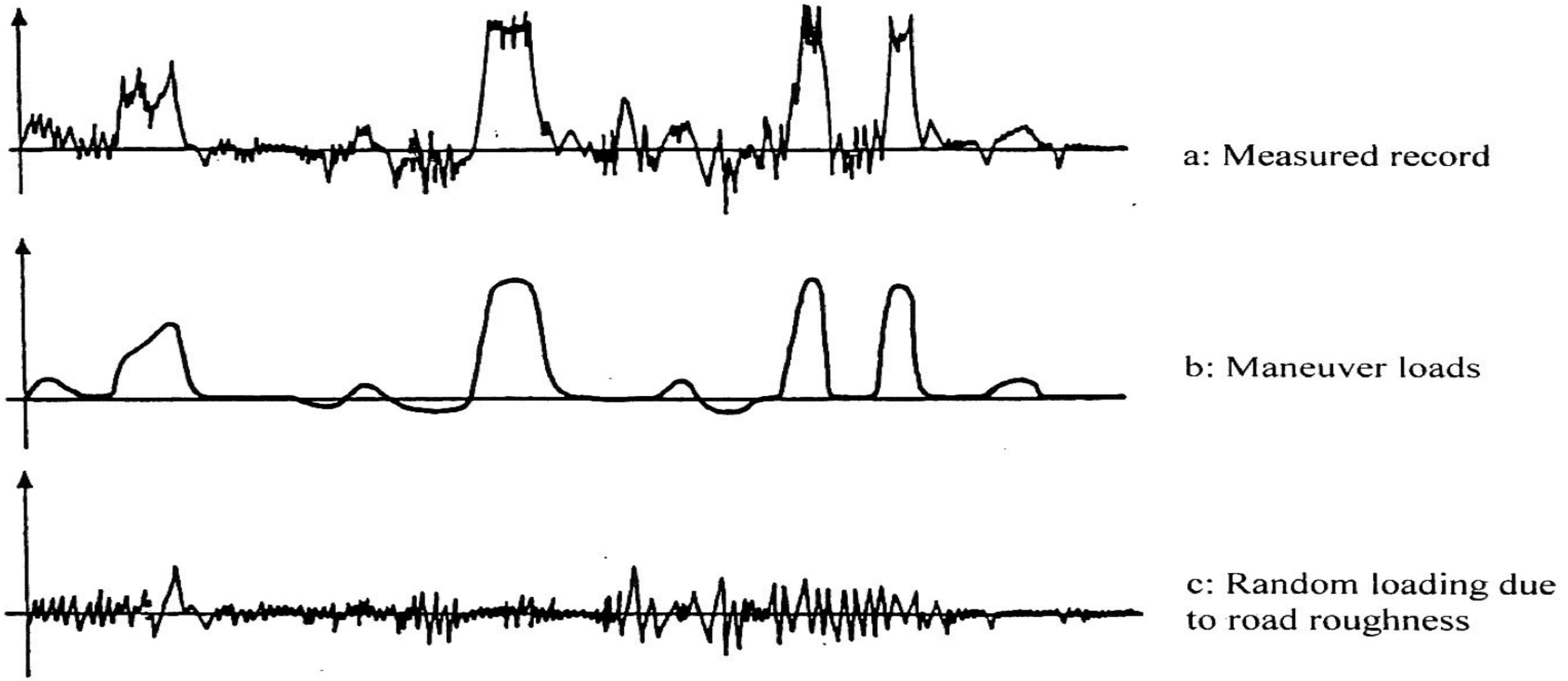
Influence line (stress in the considered detail)



Stress history

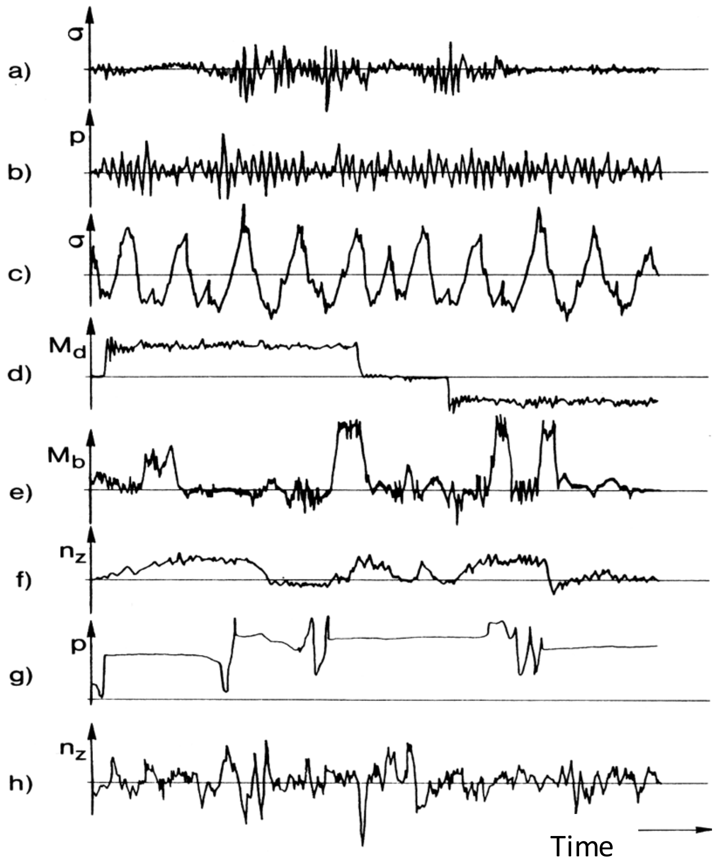


Example of variable amplitude loading (motorbike steering component)



Source: J. Schijve, Fatigue of structures and materials, 2001

Examples of variable amplitude loads



Stress in the rear axle of a car

Pressure in the condensation chamber of a reactor

Stress in the wheel of a car

Torque in the control system of a rolling mill

Bending moment in a car axle

Force at the critical point of a fighter aircraft (several flights)

Pressure in a pipe

Force at the critical point of a transport aircraft

Source: D. Radaj, M. Vormwald, Ermüdungsfestigkeit: Grundlagen für Ingenieure, 2007

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Simplification of a VA spectrum, cumulative damage law

1. Reduce the load into a series of constant amplitude cycles using a cycle counting method
2. Create a stress range histogram from the spectrum obtained under 1

Cycle counting: idea of the reservoir (or Rainflow) method

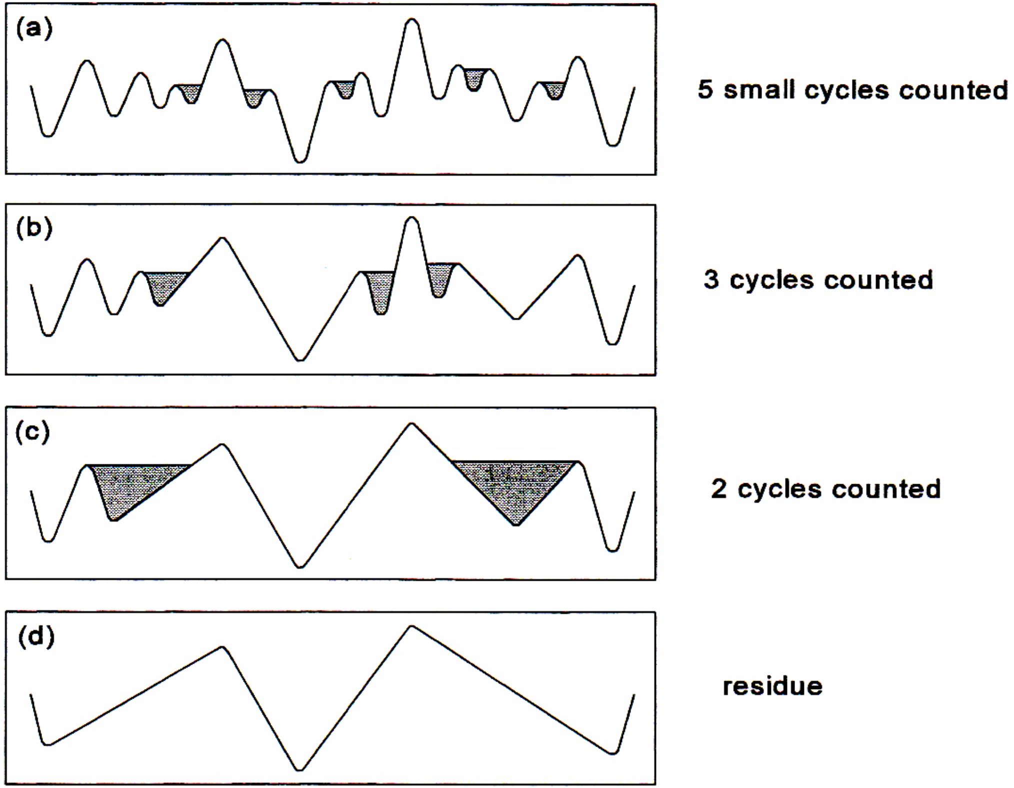
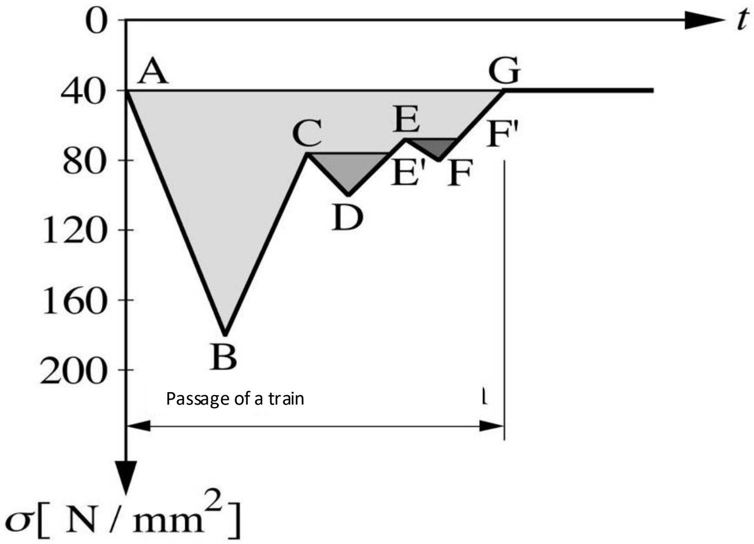


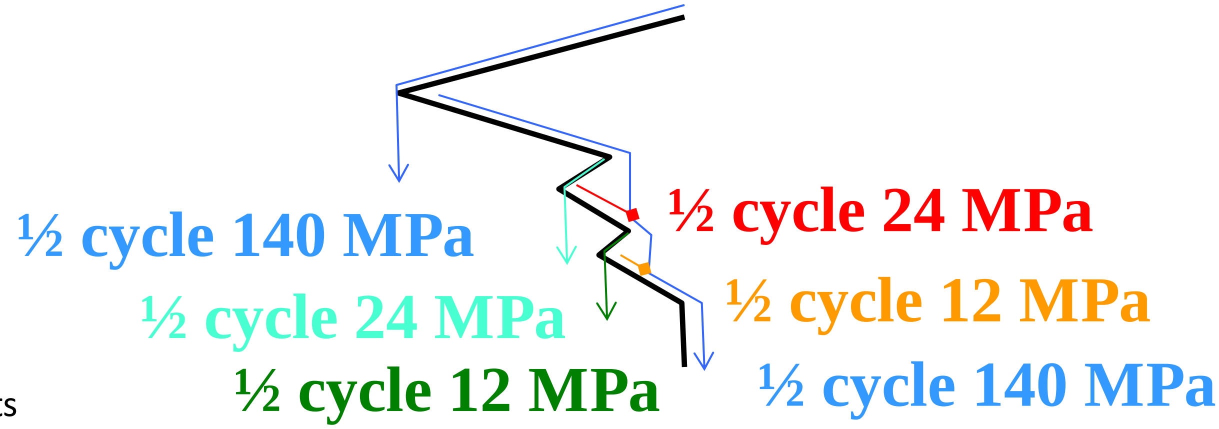
Fig.9.13: Successive Rain flow counts.

Source: J. Schijve, Fatigue of structures and materials, 2001

Reservoir counting method



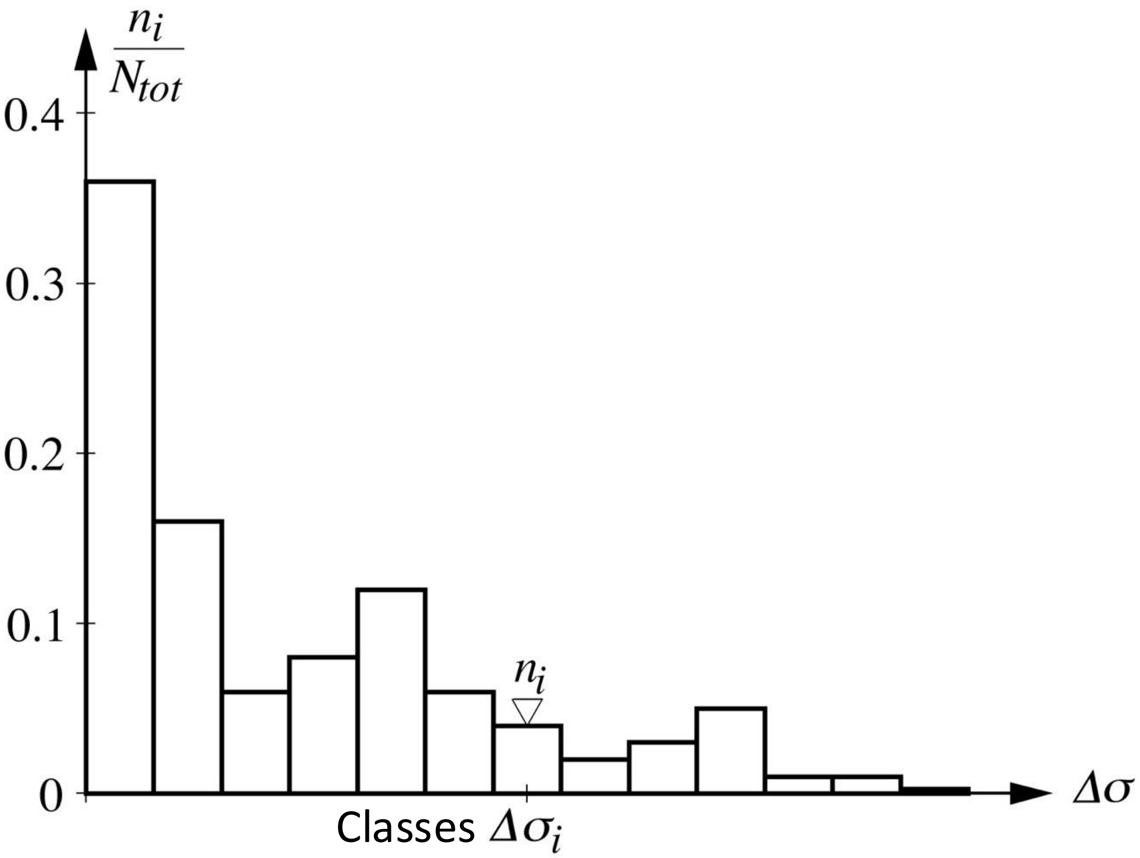
niveau-pointe	$\Delta\sigma [N/mm^2]$
$\overline{AG}$-B	140
$\overline{CE'}$-D	24
$\overline{EF'}$-F	12



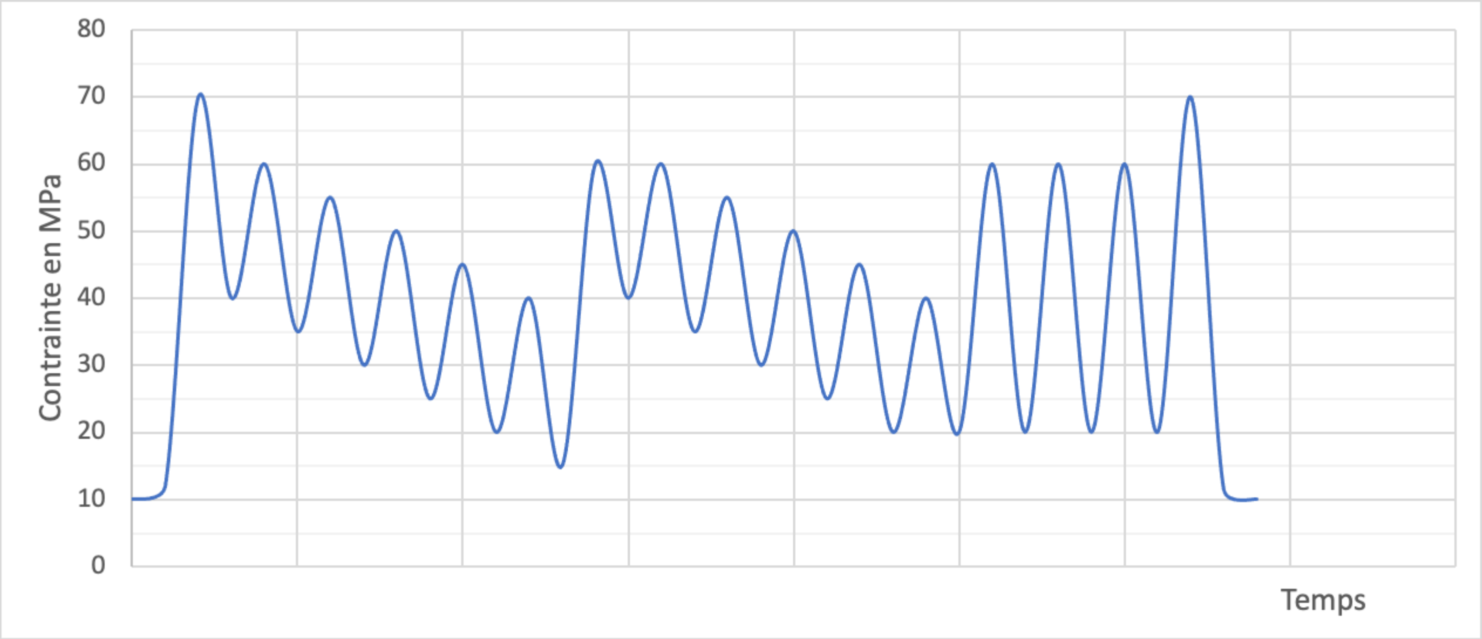
Or the equivalent,
Rainflow method
(for programming)

rainflow(x) returns cycle counts for the load time history, x

Example of results – expressed as an histogram



Example of use of the reservoir method



Simplification of a VA spectrum, cumulative damage law

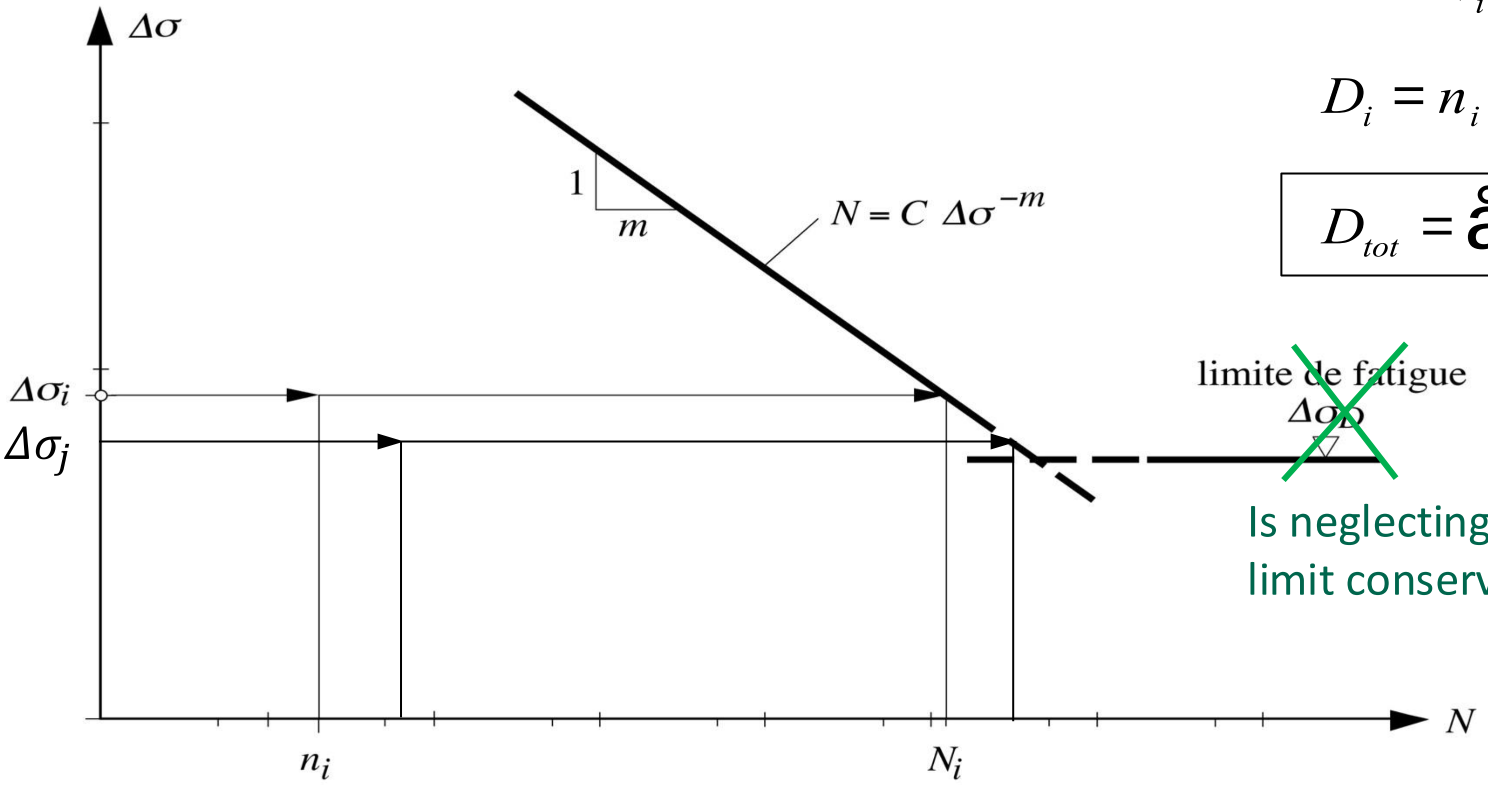
1. Reduce the load into a series of constant amplitude cycles using a cycle counting method
2. Create a stress range histogram from the spectrum obtained under 1
3. Using the appropriate resistance curve, calculate, for each stress range level, the damage value due to the number of cycles at that level using a cumulative law (Miner in general).
4.

Calculation of individual damage for n_i, $\Delta\sigma_i$

Linear accumulation law : $$d_i = \frac{1}{N_i}$$

$$D_i = n_i \times d_i = \frac{n_i}{N_i} \leq 1{,}0$$

$$D_{tot} = \sum D_i \leq 1{,}0$$



Is neglecting fatigue limit conservative ?

Calculation of damage due to n_i cycles of $\Delta\sigma_i$ **(at S-N curve design level)**

$$D_i = \frac{n_i}{N_i} \qquad\qquad N_i = C_j \cdot \left(\gamma_{Ff} \cdot \Delta\sigma_i\right)^{-m_j}$$

For part of the curve with m_j (= 3, or 5)

$$D_i = \frac{n_i}{N_i} = \frac{n_i}{C_j \cdot \left(\gamma_{Ff} \Delta\sigma_i\right)^{-m_j}}$$

And expression for C_1 from the resistance curve expressed at 2 million cycles:

$$C_1 = 2 \cdot 10^6 \cdot \left(\Delta\sigma_C / \gamma_{Mf}\right)^{m_1}$$

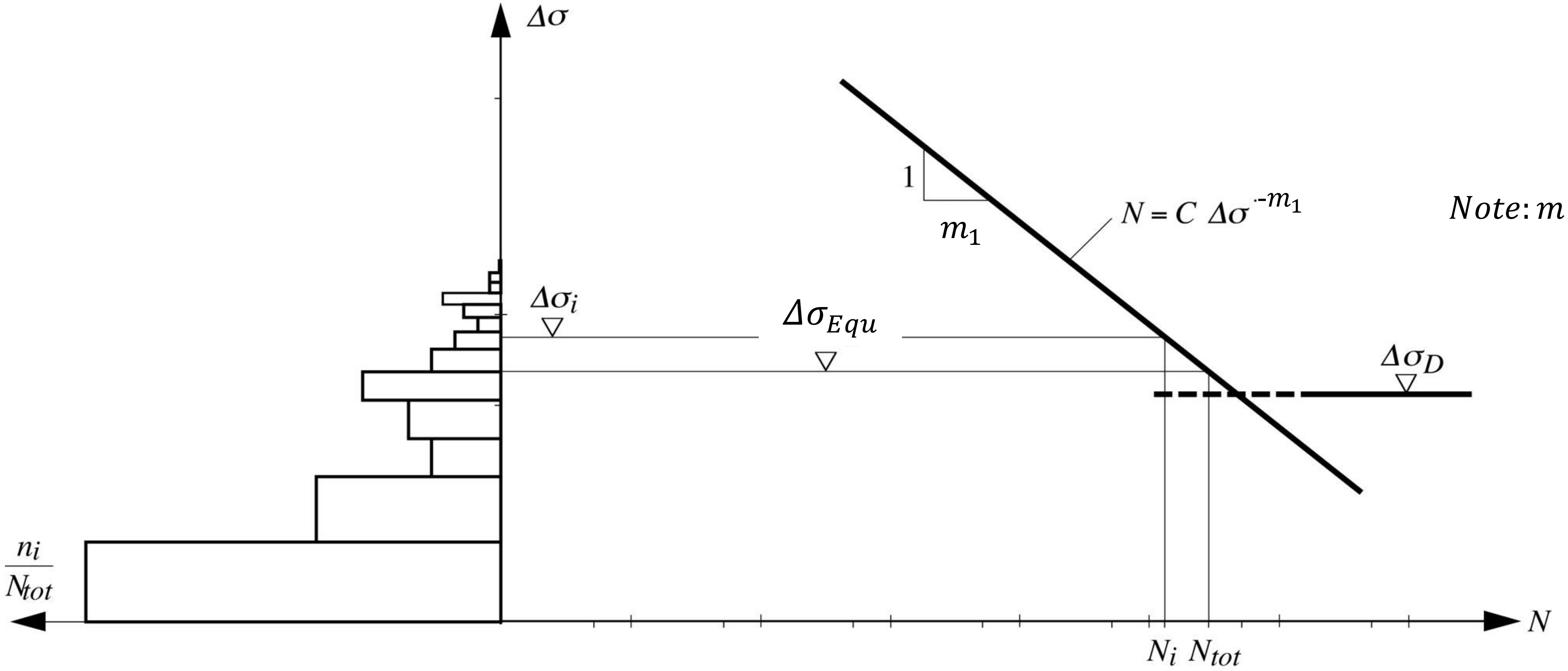
And for a level $\Delta\sigma_i < \Delta\sigma_D / \gamma_{Mf}$ (located within 2nd part of the curve):

$$C_2 = N_D \cdot \left(\Delta\sigma_D / \gamma_{Mf}\right)^{m_2}$$

Simplification of a VA spectrum, cumulative damage law

1. Reduce the load into a series of constant amplitude cycles using a cycle counting method
2. Create a stress range histogram from the spectrum obtained under 1
3. Using the appropriate resistance curve, calculate, for each stress range level, the damage value due to the number of cycles at that level using a cumulative law (Miner in general).
4. Combine the individual damages to obtain the total damage and carry out the verification

Histogram $\Delta\sigma_i$ and fatigue curve

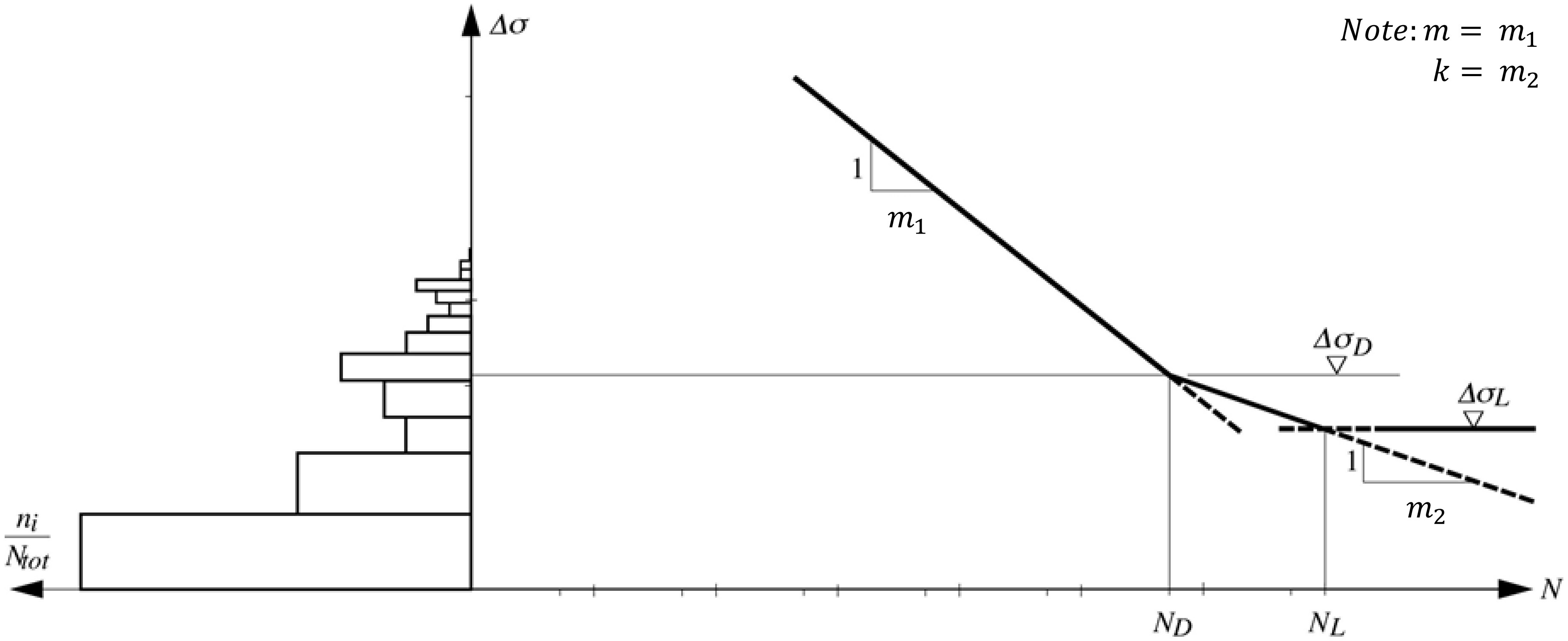


$Note: m = m_1$

$$\Delta\sigma_{Equ} = \left(\sum_{i=1}^{k} \Delta\sigma_i^{\boldsymbol{m_1}} \frac{n_i}{N_{tot}} \right)^{1/\boldsymbol{m_1}}$$

$$\Delta\sigma_{E2} = \left(\frac{1}{D_{\boldsymbol{lim}}} \frac{N_{tot}}{2 \cdot 10^6} \sum_{i=1}^{k} (\gamma_{Mf} \cdot \gamma_{Ff} \cdot \Delta\sigma_i\ )^{\boldsymbol{m_1}} \cdot \frac{n_i}{N_{tot}} \right)^{1/\boldsymbol{m_1}}$$

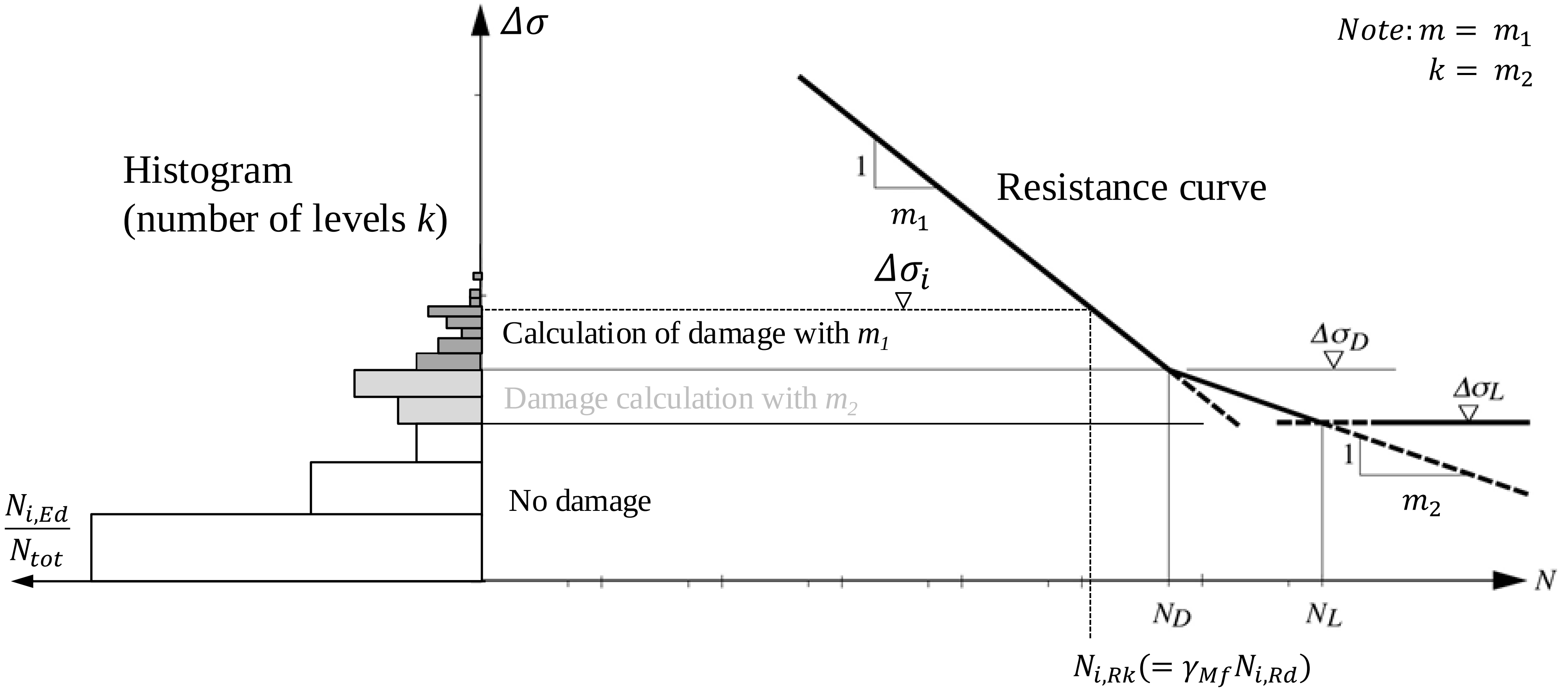
Histogram $\Delta\sigma_i$ and fatigue curve



$Note: m = m_1$

$k = m_2$

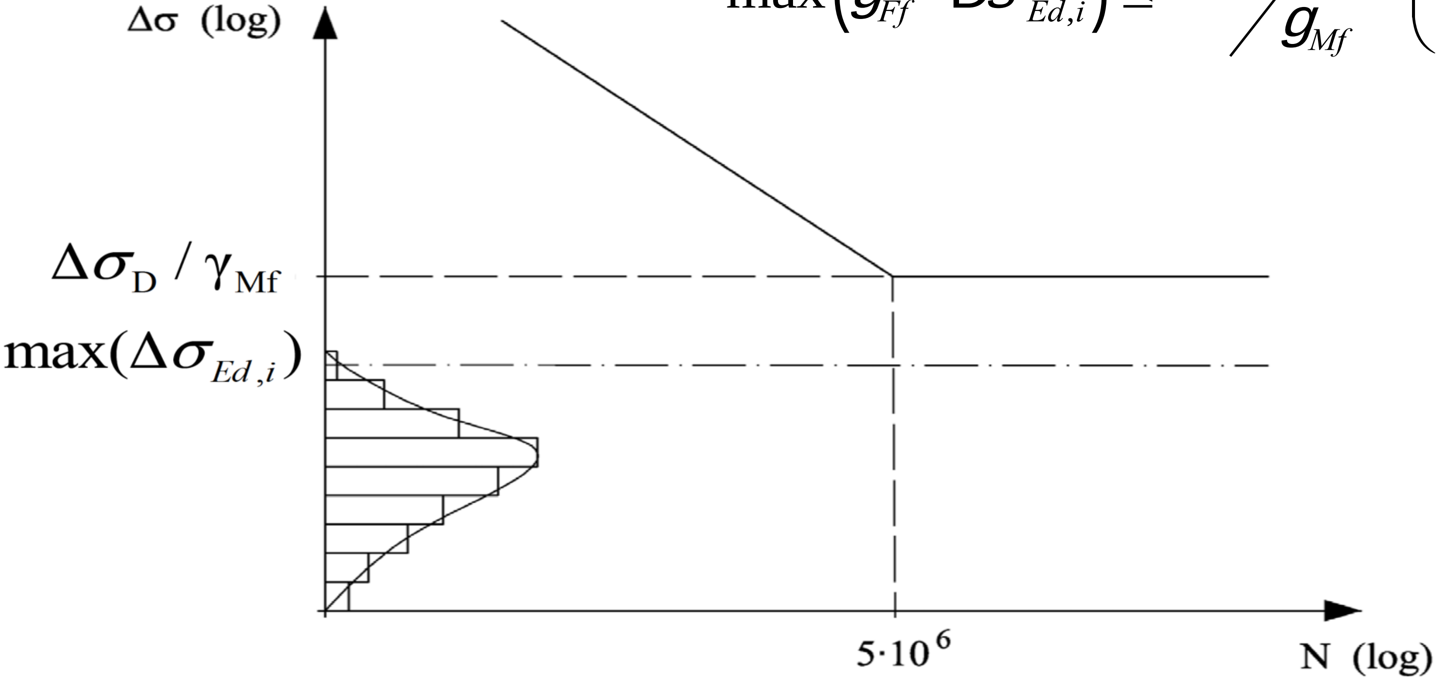
Histogram $\Delta\sigma_i$ and fatigue curve



Possible: Verification using the fatigue limit

Max of the histogram: $\max \Delta\sigma_{Ed,i}$

$$\max\left(\gamma_{Ff} \cdot \Delta\sigma_{Ed,i}\right) \leq \Delta\sigma_D / \gamma_{Mf} = \left(\frac{2}{5}\right)^{1/3} \Delta\sigma_C / \gamma_{Mf}$$



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Summary: Verification formats

1. using the fatigue limit
   (N infinite)

$$\max\left(\gamma_{Ff} \cdot \Delta\sigma_{Ed,i}\right) \leq \Delta\sigma_D / \gamma_{Mf}$$

2. using damage

$$D_{tot} = \sum \frac{n_i}{N_i} \leq D_{\lim} \quad (= 1.0)$$

3. using equivalent amplitude

$$\gamma_{Ff} \cdot \Delta\sigma_{E2} \leq \Delta\sigma_C / \gamma_{Mf}$$

4. using damage equivalence factors
   (N finite or infinite)

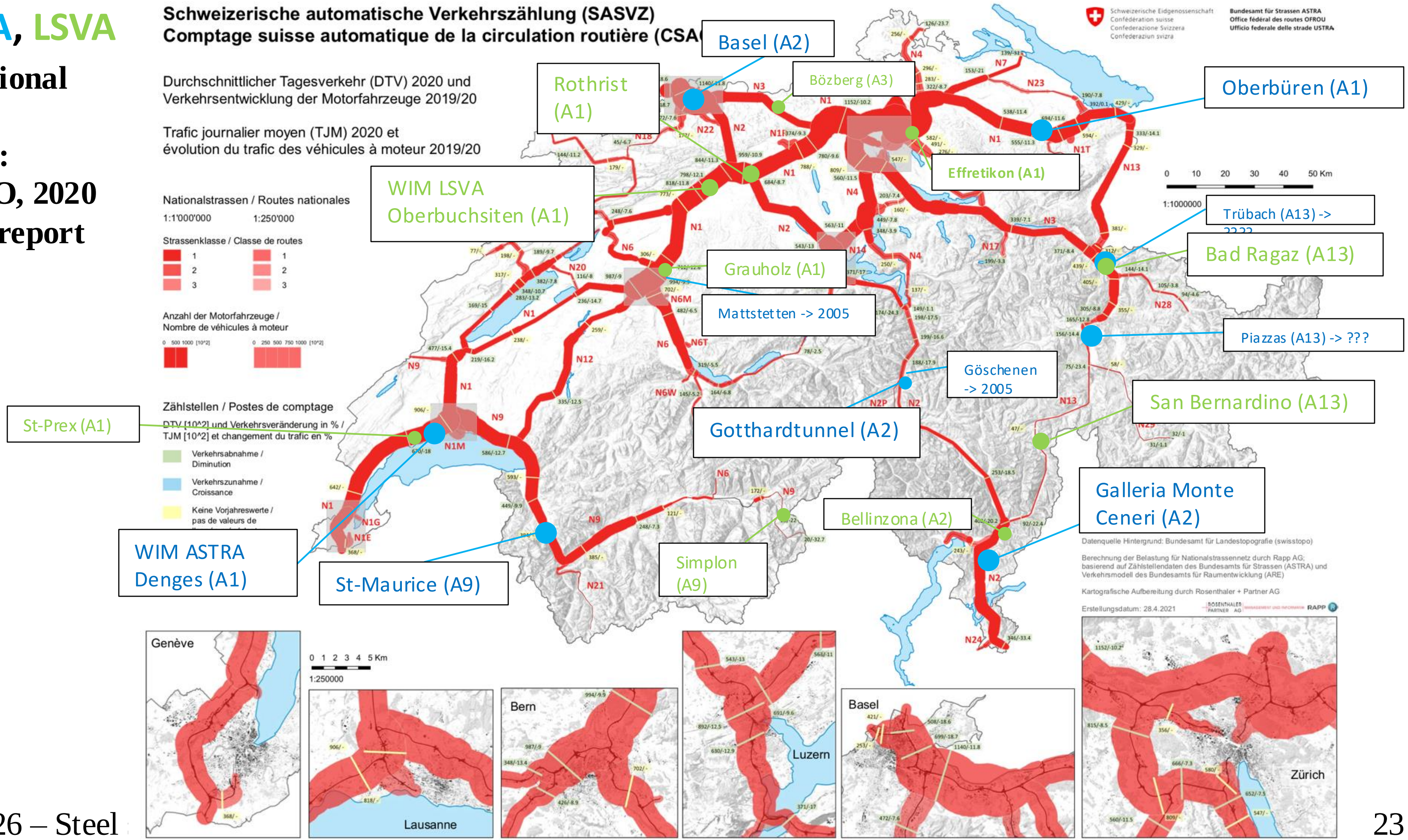
$$\lambda \cdot \Delta\sigma\left(\gamma_{Ff} Q_k\right) \leq \Delta\sigma_C / \gamma_{Mf}$$

$$\lambda_{\max} \cdot \Delta\sigma\left(\gamma_{Ff} Q_k\right) \leq \Delta\sigma_C / \gamma_{Mf}$$

Switzerland: traffic volume and WIM stations

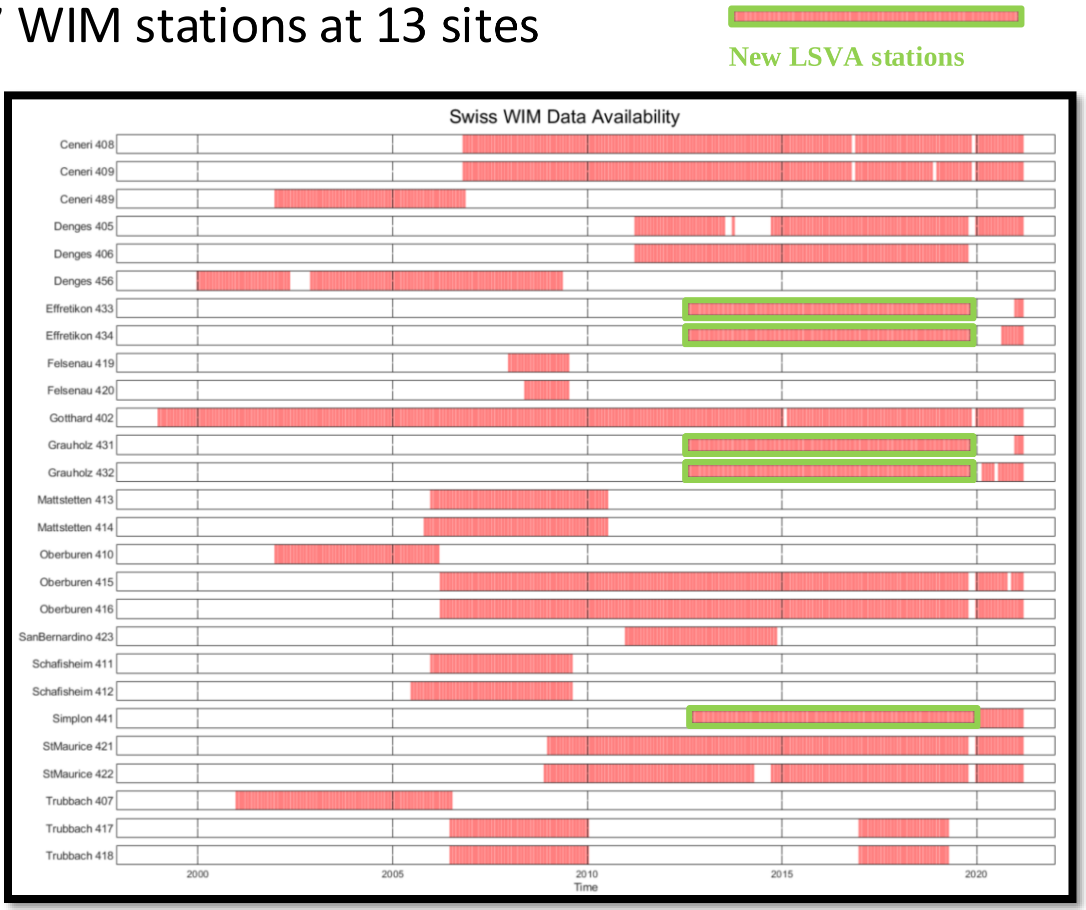
ASTRA, LSVA

On national roads
Source: FEDRO, 2020 traffic report

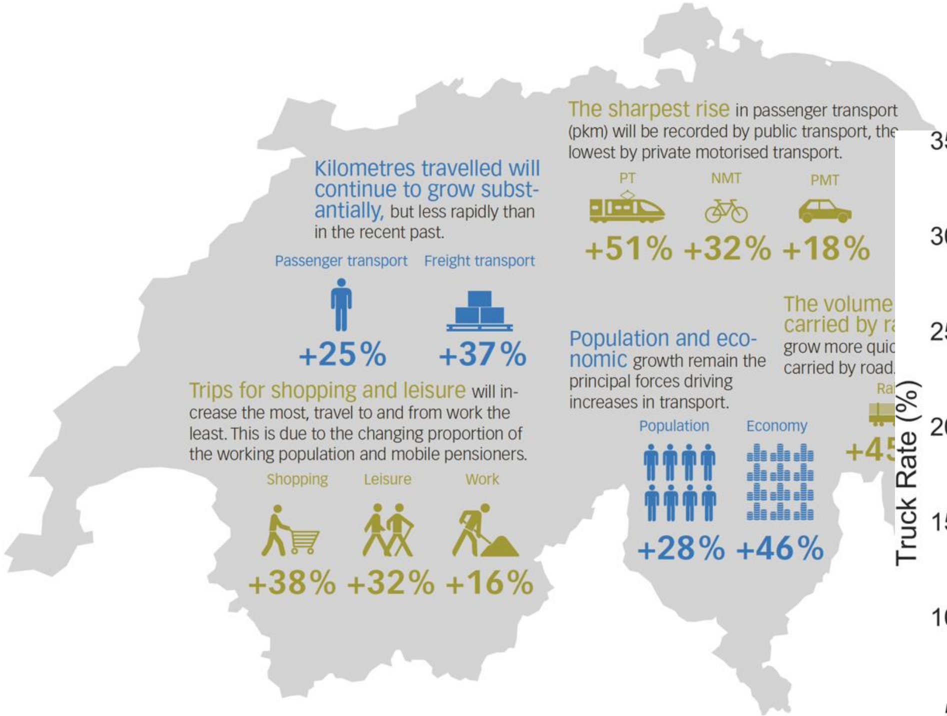


Available WIM data

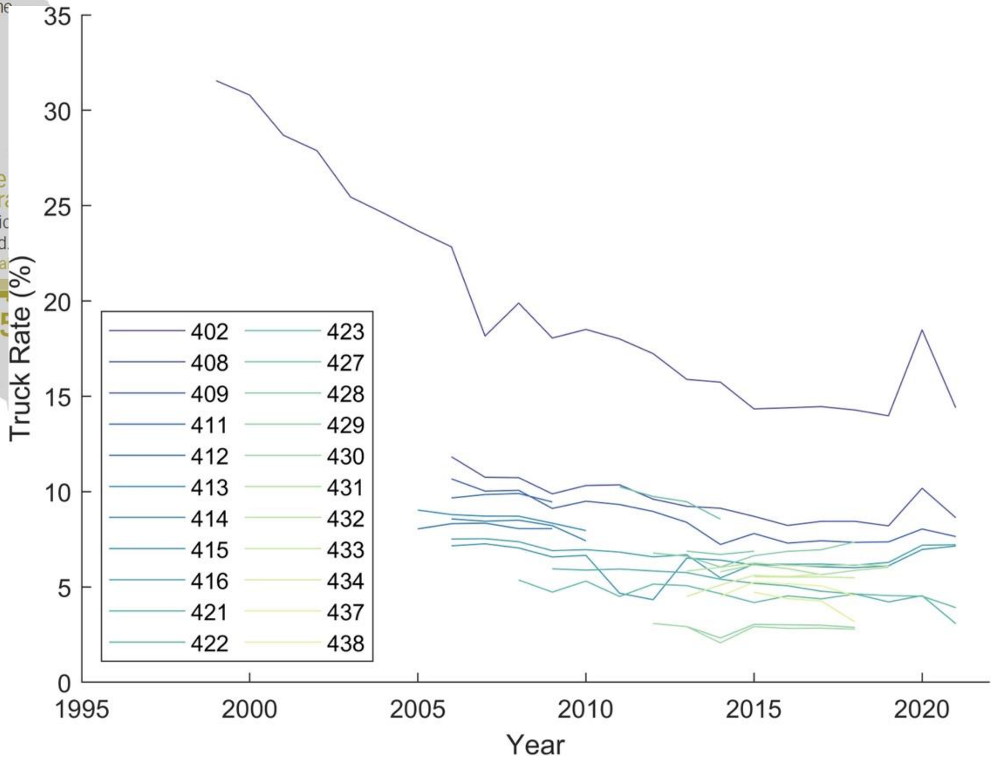
- Data from 27 WIM stations at 13 sites



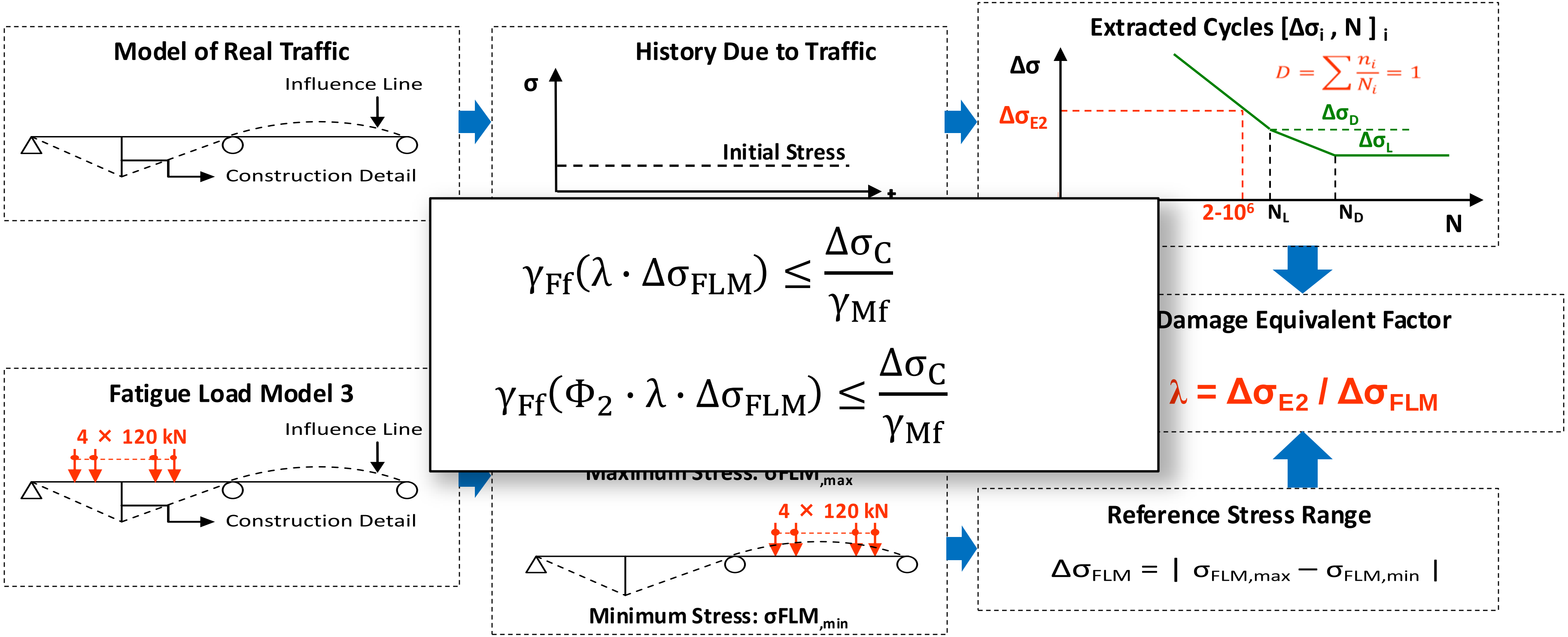
Switerland: transport outlook 2040



Source: Federal Office for Spatial Development ARE, August 2016



Evaluation of Eurocode Damage Equivalent Factor Based on Traffic Simulation



$$\gamma_{Ff}(\lambda \cdot \Delta\sigma_{FLM}) \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$$\gamma_{Ff}(\Phi_2 \cdot \lambda \cdot \Delta\sigma_{FLM}) \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

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Simplified fatigue verification format: damage equivalent factor concept

$$\gamma_{Ff} \cdot \Delta\sigma_{E2} \leq \Delta\sigma_C / \gamma_{Mf}$$

- γ_{Ff}: Load factor
- $\Delta\sigma_{E2}$: Actions
- $\Delta\sigma_C$: Category = resistance 2 million cycles
- γ_{Mf}: Resistance factor

$$\Delta\sigma_{E2} = \lambda \cdot \Delta\sigma(Q_k)$$

- λ depends on the fatigue load model (SIA or FLM3)

- Same format for $\Delta\tau$, same λ

$$\gamma_{Ff} \cdot \Delta\tau_{E2} \leq \Delta\tau_C / \gamma_{Mf}$$

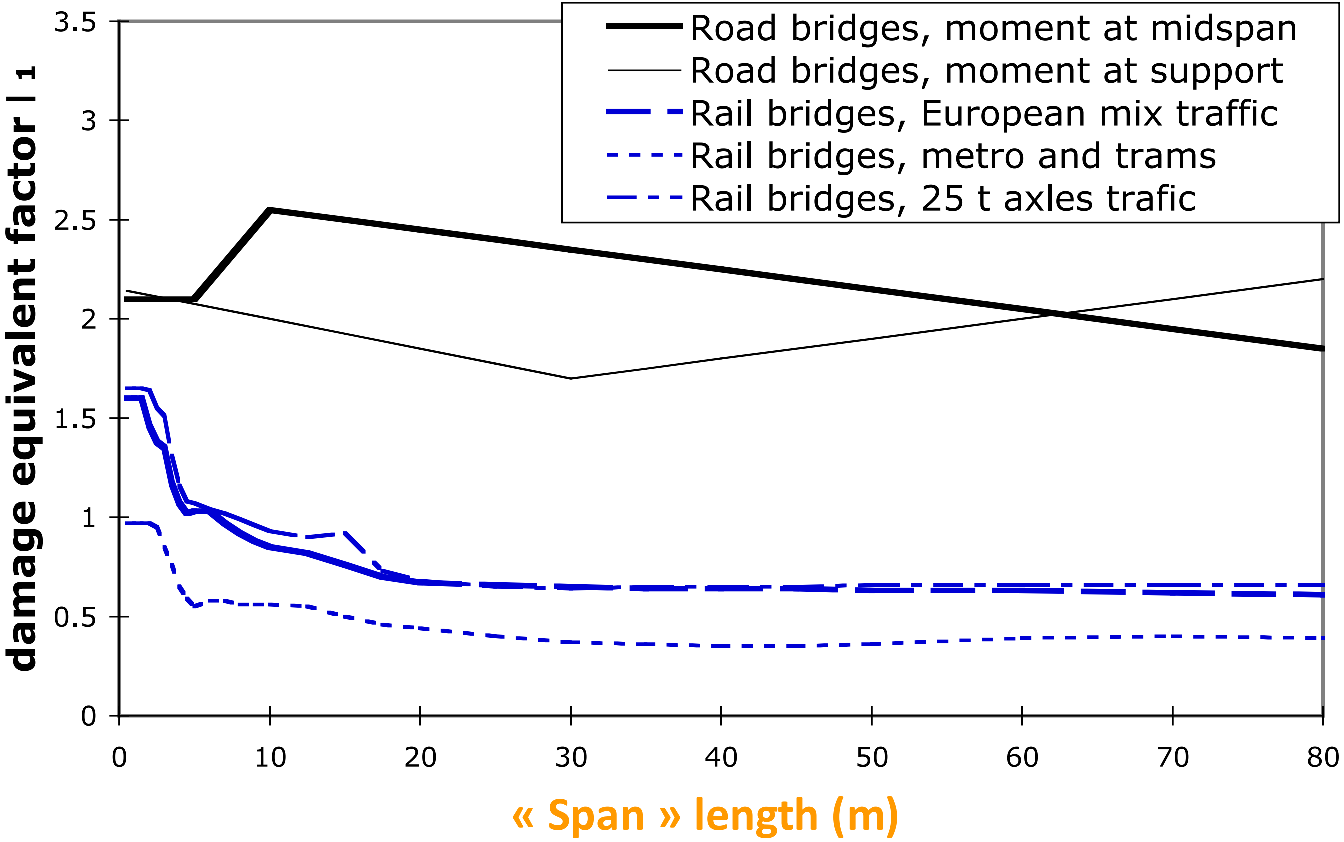
EN 1993-2 and SIA 263: 2013: Partial damage equivalent factors

$$/ = /_{1} \times /_{2} \times /_{3} \times /_{4} \; £ \; /_{max}$$

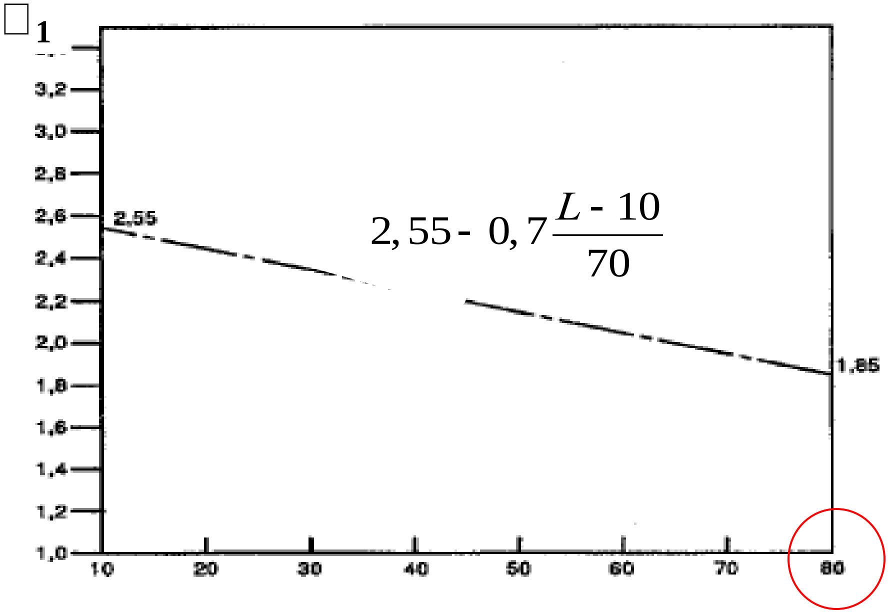
With:

λ_1	effect of traffic as a function of the determining length, or of determining influence surface
λ_2	effect of traffic volume, in weight and nb, if different then N_0 from code
λ_3	effect of length of service, if different then code (for cst traffic)
λ_4	effect of heavy traffic on other lanes (if several lanes)
λ_{max}	maximum value, taking into account the fatigue limit (according to λ of the determining length)

Damage equivalent factor λ_1 (EN1993-2)



Damage equivalent factor λ_1 (EN1993-2)



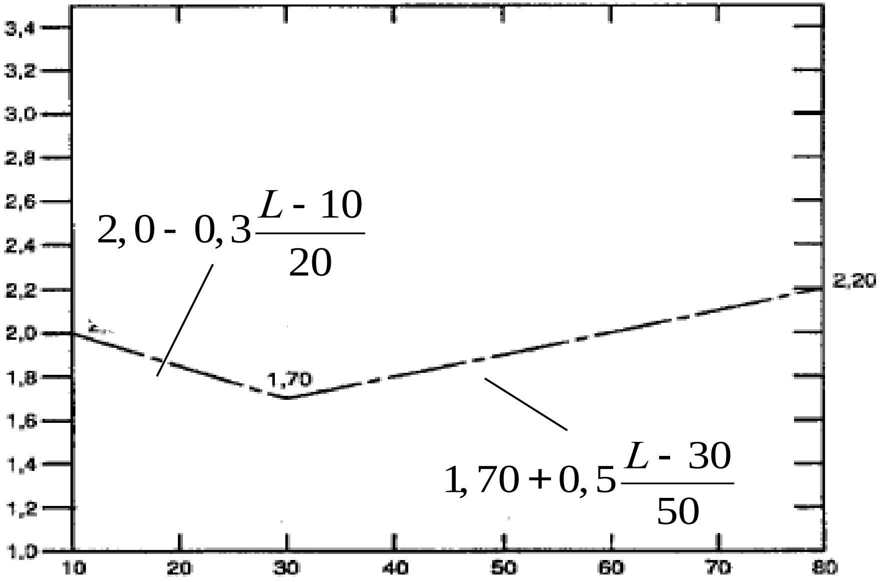
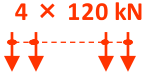
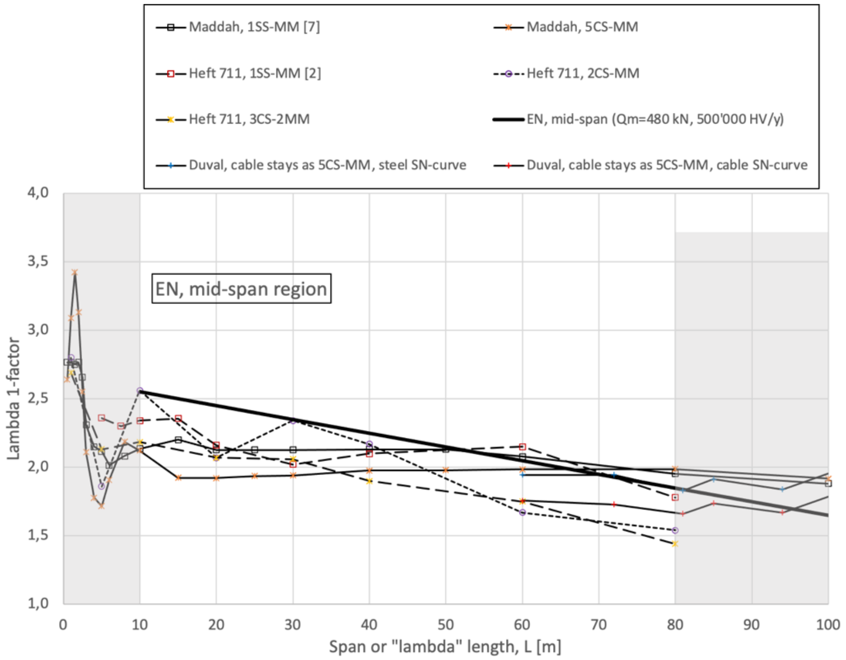


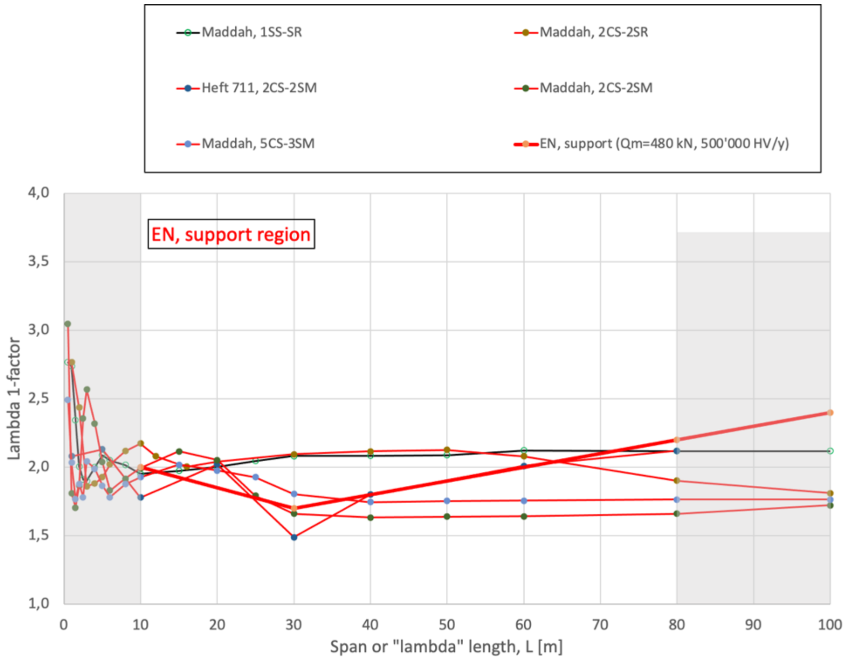
Figure 9.5 — λ_1 **for bending moments in road bridges**



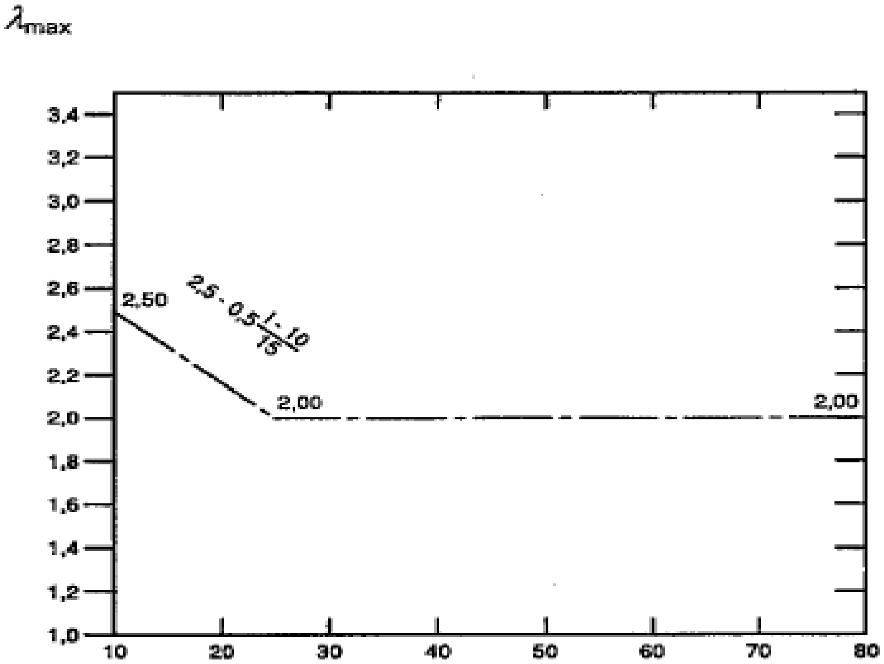
$N_0 = 0.5 \cdot 10^6$ trucks

Comparison between λ_1 curves and simulations



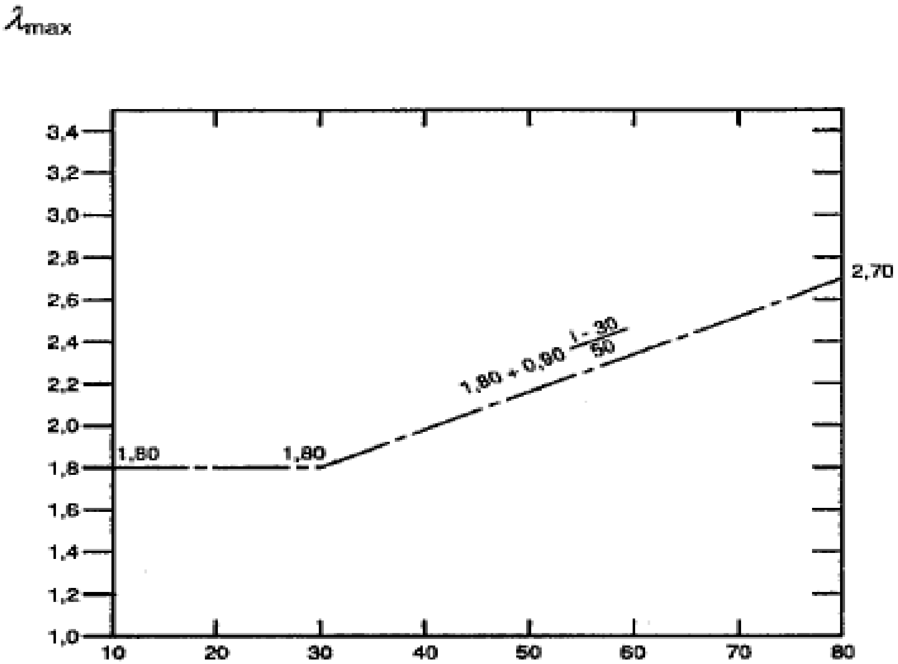


Damage equivalent factor λ_{max} (EN1993-2)



Span L [m]

At mid-span



Span L [m]

At support

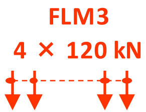


Figure 9.6 — λ_{max} for bending moments in road bridges

Partial fatigue equivalent factors λ_2 and λ_3

Stress range verification:

$$\lambda \cdot \Delta\sigma_{Ed} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

With $N \cdot \Delta\sigma^m = cte$

We have the following proportionalities $\lambda \propto \Delta\sigma; N^{1/m}$

And expressions for lambda partial factors:

$$\lambda_2 = \frac{Q_m}{Q_0}\left(\frac{N_{obs}}{N_0}\right)^{1/m_2}$$

With:

$m_2 = 5$ (steel, SIA 263)

Q_m = average weight of heavy vehicles on slow lane

Q_0 = 480 kN (reference weight)

$N_0 = 0.5 \cdot 10^6$ load cycles on the slow lane

$$Q_m = \left(\frac{\sum n_i Q_i^{m_2}}{\sum n_i}\right)^{1/m_2}$$

$$\lambda_3 = \left(\frac{T_{Ld}}{100}\right)^{1/m_2}$$

With:

T_{Ld} = expected service life (in years)

Note: concrete bridges with specific values (because of specific S-N curves)

Partial fatigue correction factor λ 4

For road bridges:

$$\lambda_4 = \left[1 + \frac{N_2}{N_1}\left(\frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}}\right)^{m_2} + \frac{N_3}{N_1}\left(\frac{\eta_3 Q_{m3}}{\eta_1 Q_{m1}}\right)^{m_2} + \ldots + \frac{N_k}{N_1}\left(\frac{\eta_k Q_{mk}}{\eta_1 Q_{m1}}\right)^{m_2} \right]^{1/m_2} \geq 1{,}0$$

With:

k nb of lanes carrying heavy traffic

N_j nb of heavy vehicles per year on lane j

Q_{mj} average weight of heavy vehicles on lane j (note: $Q_{m1} = Q_m$)

η_j value of the transverse distribution line at the centre of track j which produces the stress range, with a positive sign

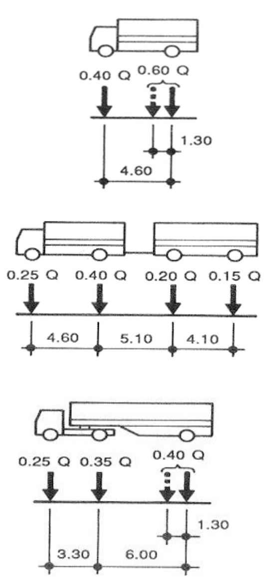
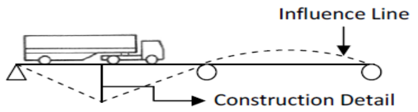
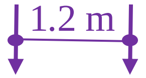
For railway bridges:

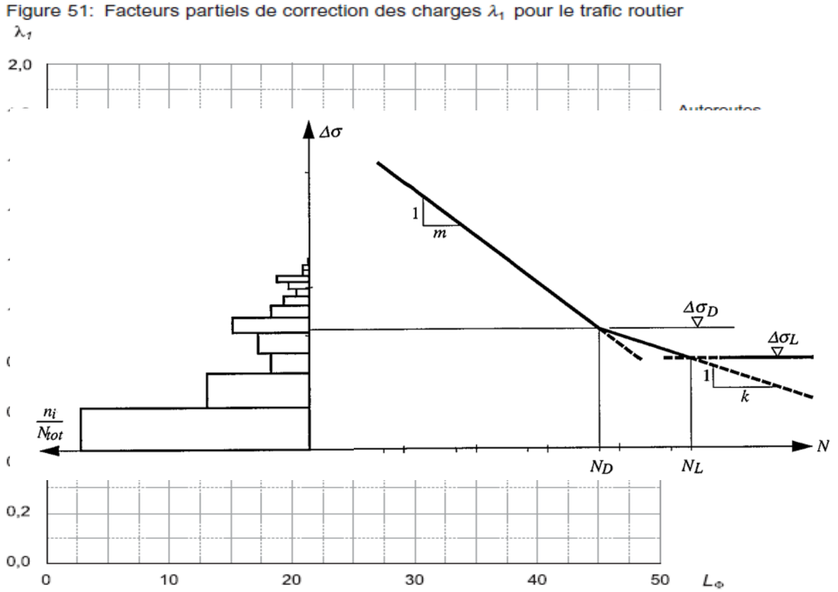
$$\lambda_4 = \left(n + [1 - n]\left[a^{m_2} + (1 - a)^{m_2} \right] \right)^{1/m_2} \leq 1{,}0$$

$a = \Delta\sigma_1 / \Delta\sigma_{1+2}$

n portion of traffic together (waiting or crossing) on the bridge, by default 12%

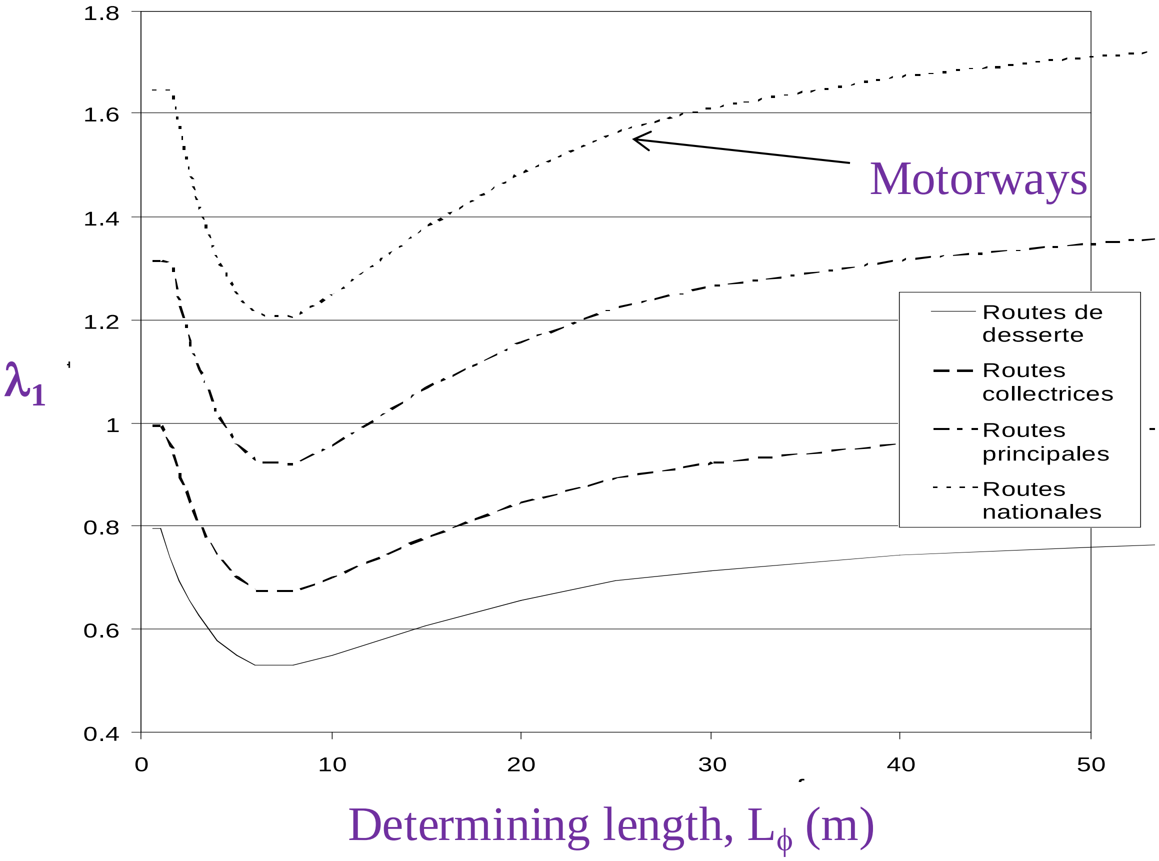
SIA 261/263: calibration assumptions λ_1 for road bridges

3 types of heavy veh. (EN min. 5 types)	Passage of sequence of vehicles
	
	Fatigue Load model SIA 261, Q_{fat}
	
	Fatigue verification
	$\underbrace{\lambda\,\Delta\sigma(Q_{fat})}_{\Delta\sigma_{E2}} \le \Delta\sigma_c\,/\gamma_{Mf}$

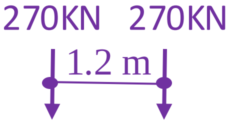


Note: factor λ_1 given for heavy traffic per year and slow lane. E.g. for Swiss national roads (motorways), 1'400'000 vehicles/year/slow lane (in Europe, Cat. 1 = 2 million).

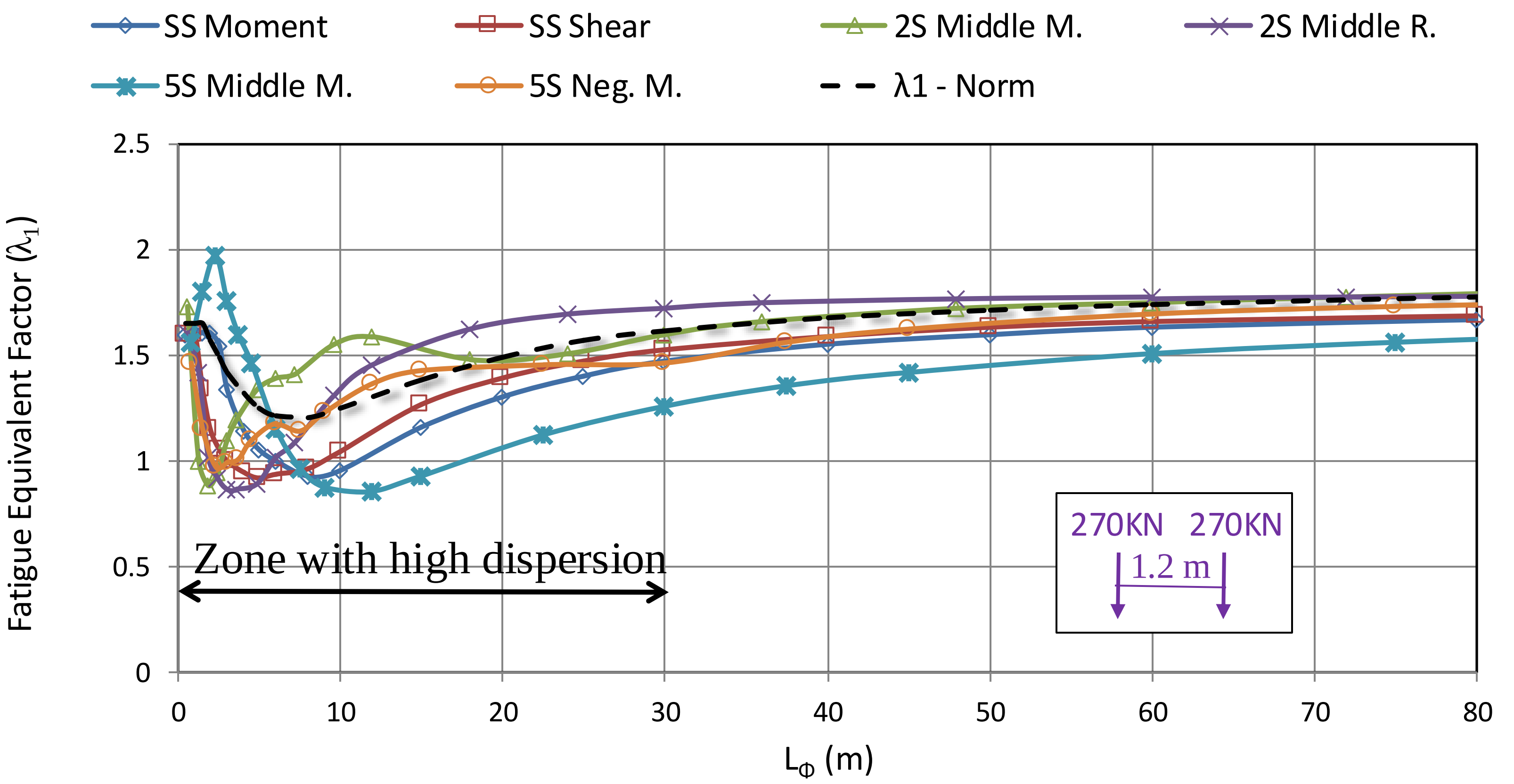
SIA 263: damage equivalent factor λ_1



SIA fatigue load model



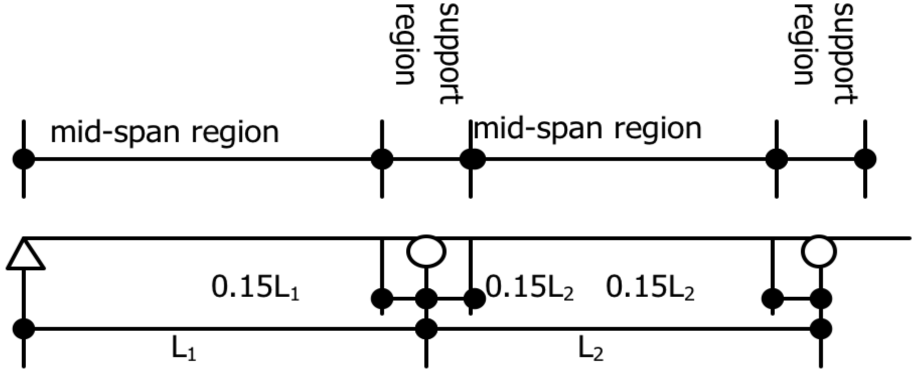
Dispersion on λ_1 depending on the static system, Determining length



EN1993-2 and SIA261, Annex F: calculation of determining length (code/FLM dependent, can be fct of influence line)

For stresses resulting from bending moments:

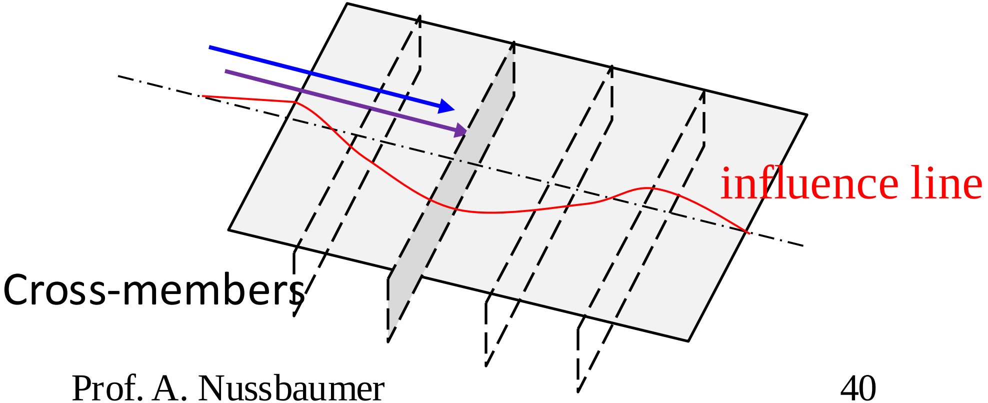
- For a simply supported span, span L_i
- For continuous spans in mid-span regions, span length L_i of span under consideration
- For continuous spans in support regions, mean of 2 adjacent spans L_i and L_j to that support
- For cross-girders supporting stringers (or rail bearers), sum of 2 adjacent spans of stringers (rail bearers) $L_i + L_j$ carried by cross-girder
- Definition of regions:



EN1993-2 and SIA: calculation of determining length

For stresses resulting from bending moments:

- ...
- For a deck plate or slab supported only by transverse beams or cross-ribs (no longitudinal members), the length of the influence line used to calculate the deflection of the plate, ignoring any part that indicates upward deflection. The same applies to transverse beams, cross-ribs, themselves. In rail bridges, the stiffness of the rails on the load distribution must be considered. For transverse beams, cross-ribs, spaced not more than 750 mm apart, this may be taken as 2 × cross-member spacing + 3 m



EN1993-2 and SIA: calculation of determining length

For shear for simply supported and continuous spans:

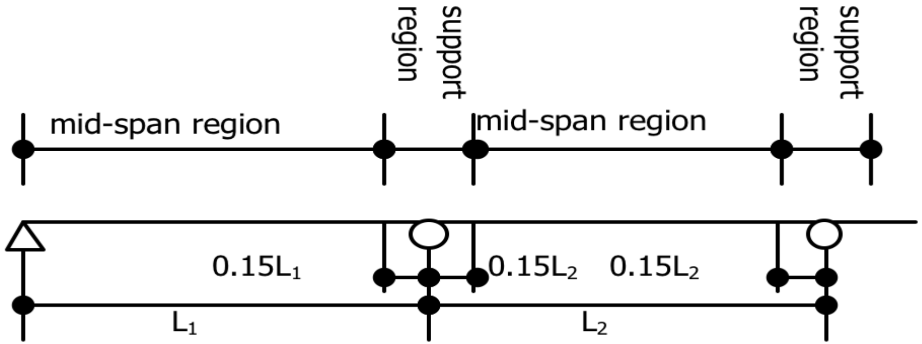
- For support regions, span L_i with section under consideration
- For mid-span regions, span of $0.4 \cdot L_i$ with section under consideration

For reactions:

- For end support, span under consideration L_i
- For intermediate supports, sum of 2 adjacent spans $L_i + L_j$

Arch bridges:

- For hangers, 2 × length of hanger under consideration
- For arch, ½ span



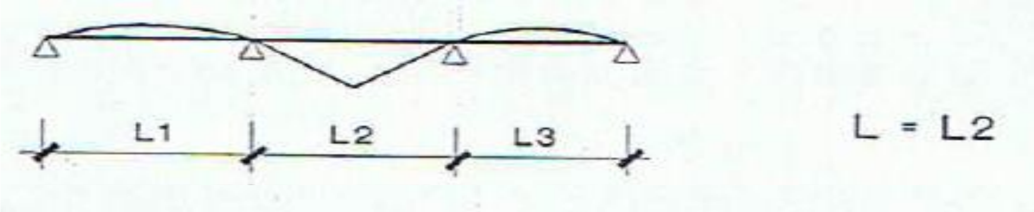
EN1993-2 and SIA: calculation of determining length

Untreated cases:

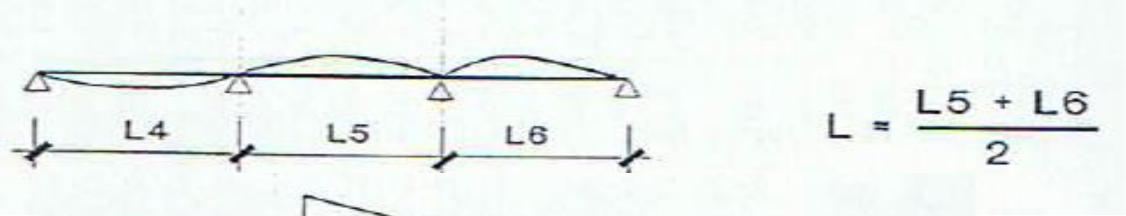
- Important, even if approximate, respect the shape of the Influence Line (one, two, etc. bumps, of same sign or not)
- Determine I.L. for each element/internal force
- If possible, analogy with I.L. of a simple beam to fix the determining length
- For railway bridges, if there are two or more I.L. zones, safer to use the shortest influence length.

Examples of calculation of determining length

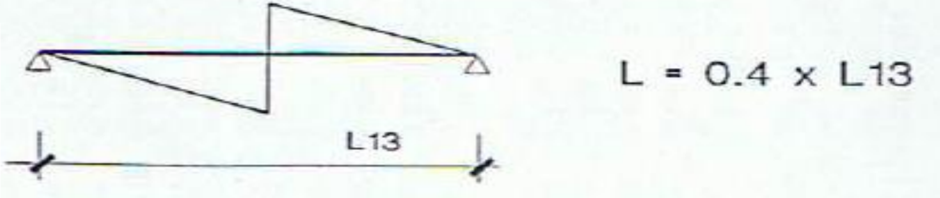
M mid-span of 2nd span



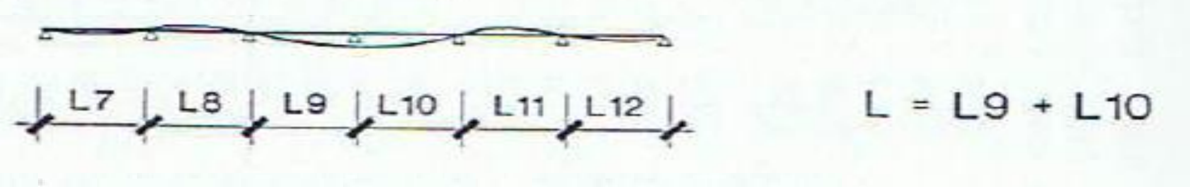
M on 3rd support



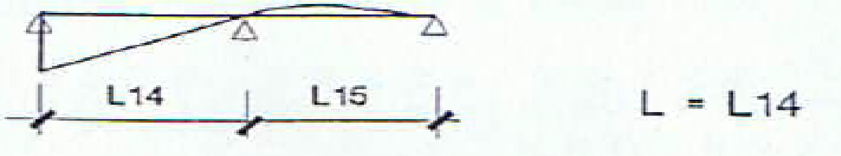
V simple beam



R on 4th support

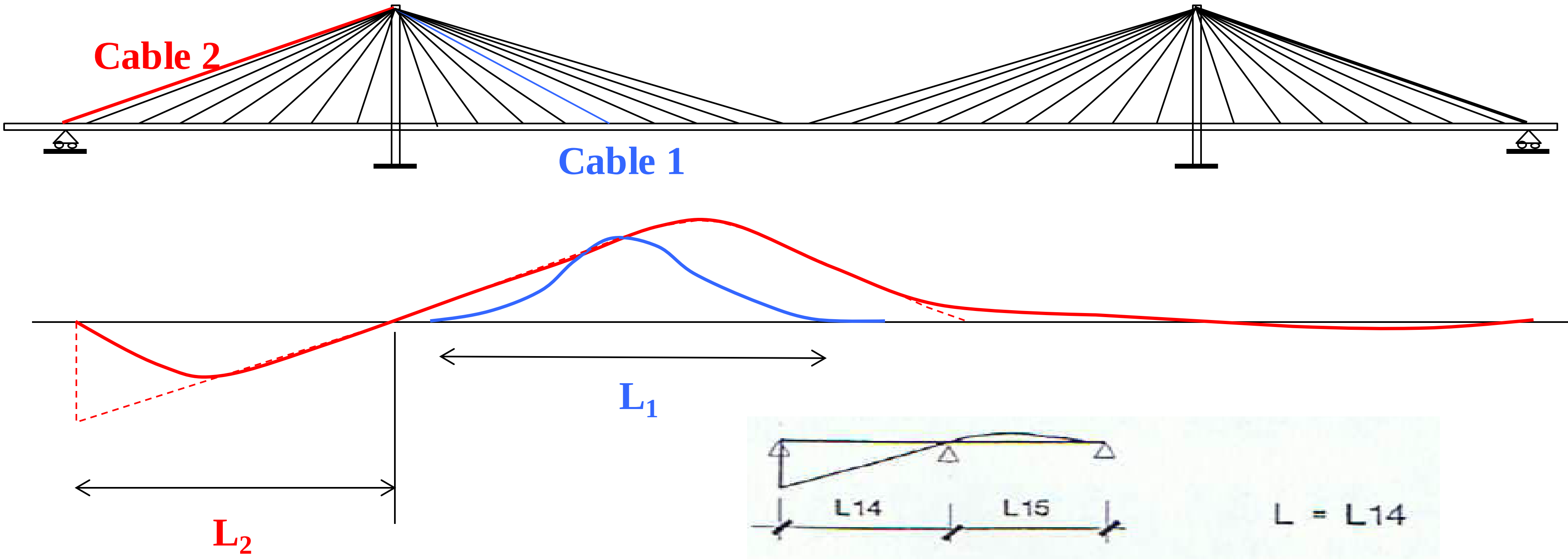


R on abutment



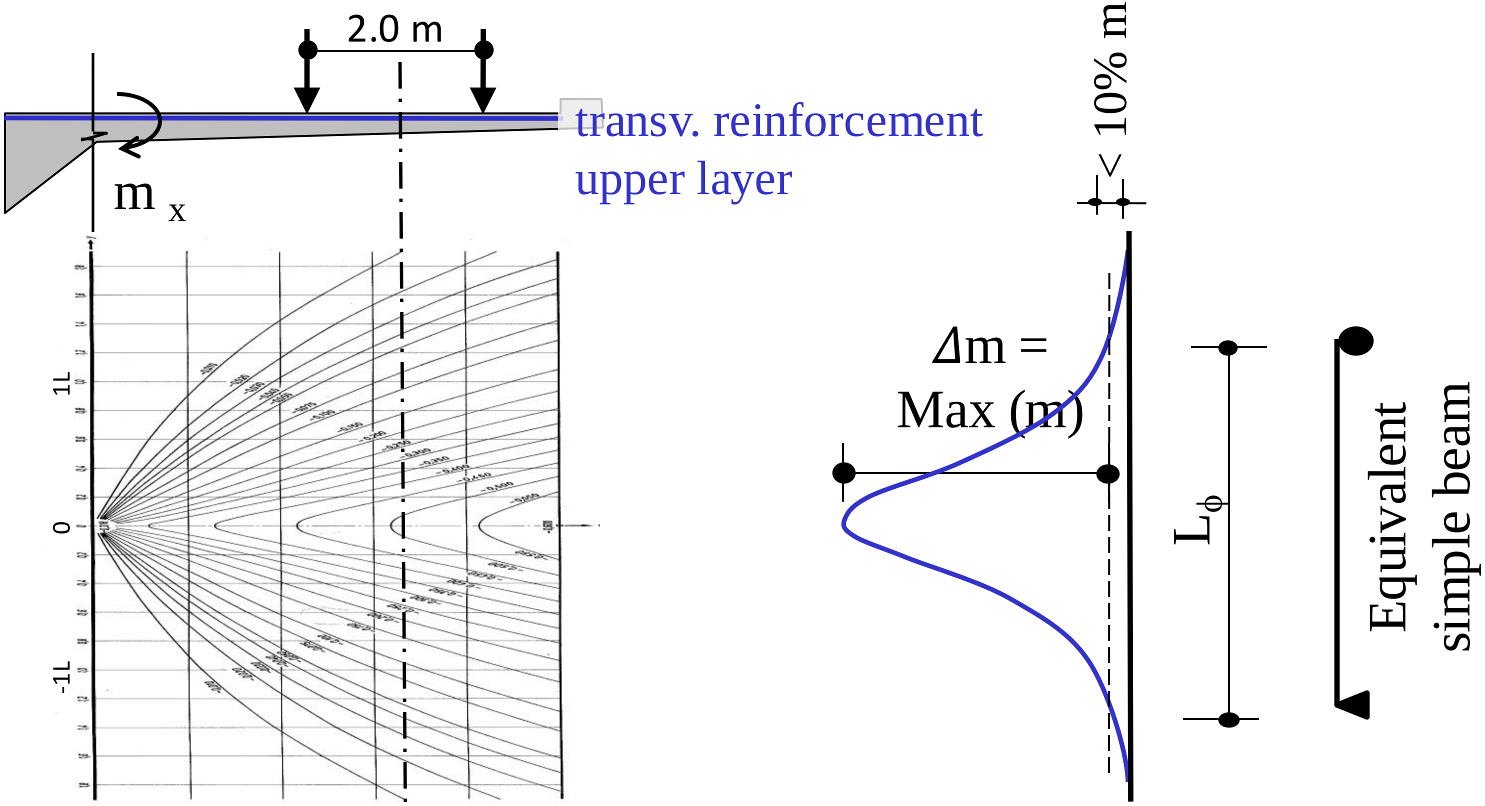
Examples of calculation by analogy of determining length

- Axial force on cable-stayed bridge cable
- By analogy, ressembles a support reaction
- Lambda value taken from graph: support region



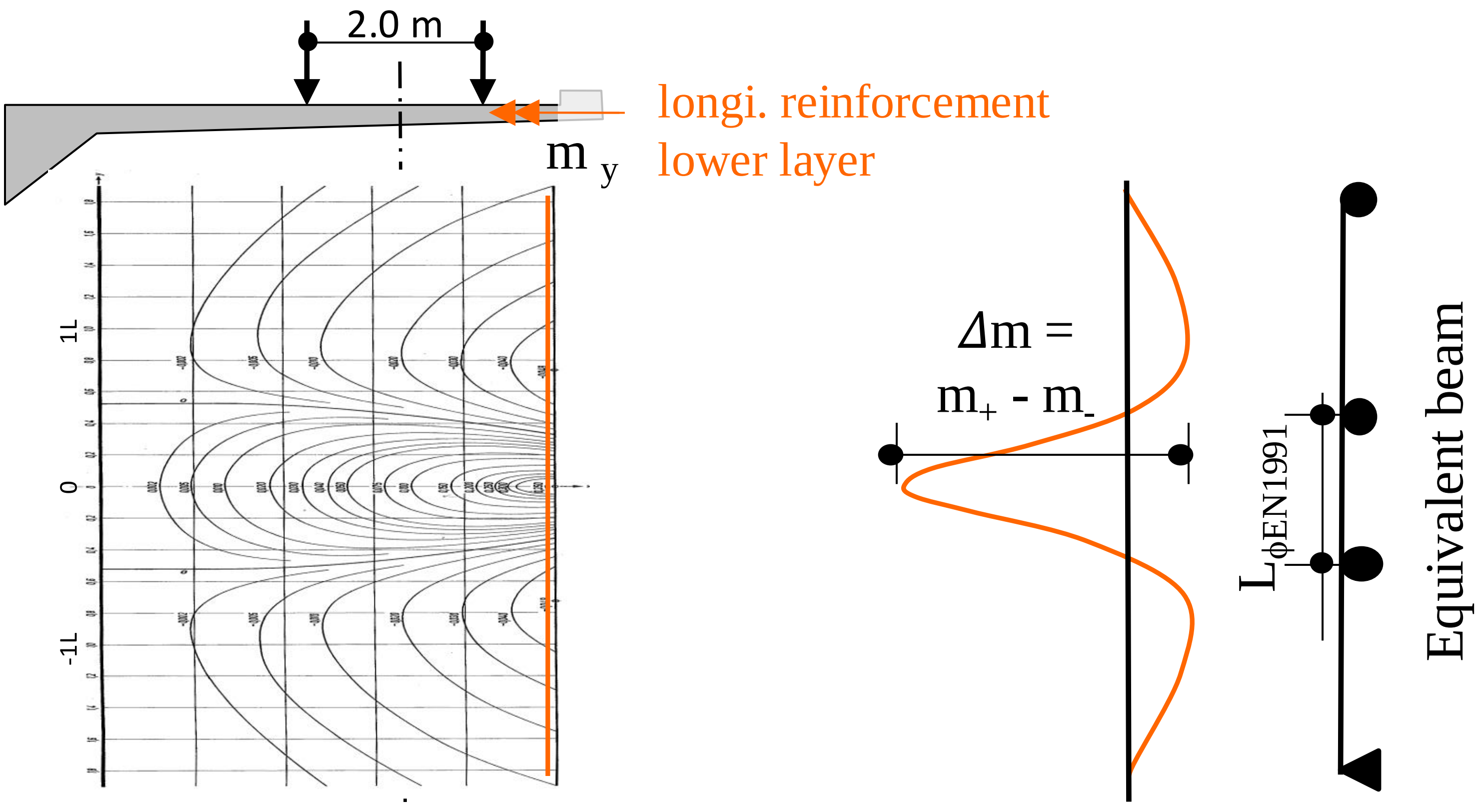
Calculation by analogy of determining length L_Φ, bridge slab

- Area of influence moment m_x on cantilever support



Calculation by analogy of determining length L_{Φ}, bridge slab

- Area of influence moment m_y end of cantilever



Resistance factor γ_{Mf} (current EN 1993-1-9 and SIA)

- Not a single value
- Based on possibility to perform visual inspections (values may be adapted wrt inspection intervals and methods)
- Prerequisite, choice of steel quality according to EN 1993-1-10
- Redundancy taken into account within both damage tolerant concept and consequences of failure

	Consequences of failure	
	Low	**Important**
Inspection and repair: Possible ←→ damage tolerant concept	**1.00**	**1.15**
Not possible ←→ safe life concept	**1.15**	**1.35**

Resistance factor γ_{Mf} (NEW EN 1993-1-9: 2027)

- Still a proposition, not yet voted by CEN
- Explicit link between partial factor values and reliability requirements:
  - Low consequence ⇔ Class of Consequences CC1 according to EN 1990
  - Medium consequence ⇔ CC2
  - High consequence ⇔ CC3

Table 5.1 (NDP) — Recommended values of the partial factors for fatigue resistance γ_{Mf}

Design concept	Consequence of failure		
	Low consequence	Medium consequence	High consequence
Safe life	1,15	1,25	1,35
Damage tolerant	1,00	1,15	1,25

Design concepts requirements

Select appropriate constructional details, materials and stress levels to ensure sufficient reliability level.

For safe life concept : at end of design service life (no need for inspections).
To apply where local formation of cracks in one constructional detail could rapidly lead to failure of a structure or one of its parts.

Examples: single anchor cable, single bolt connection, some details in case of simple spans twin-beam systems, etc.

Design concepts requirements

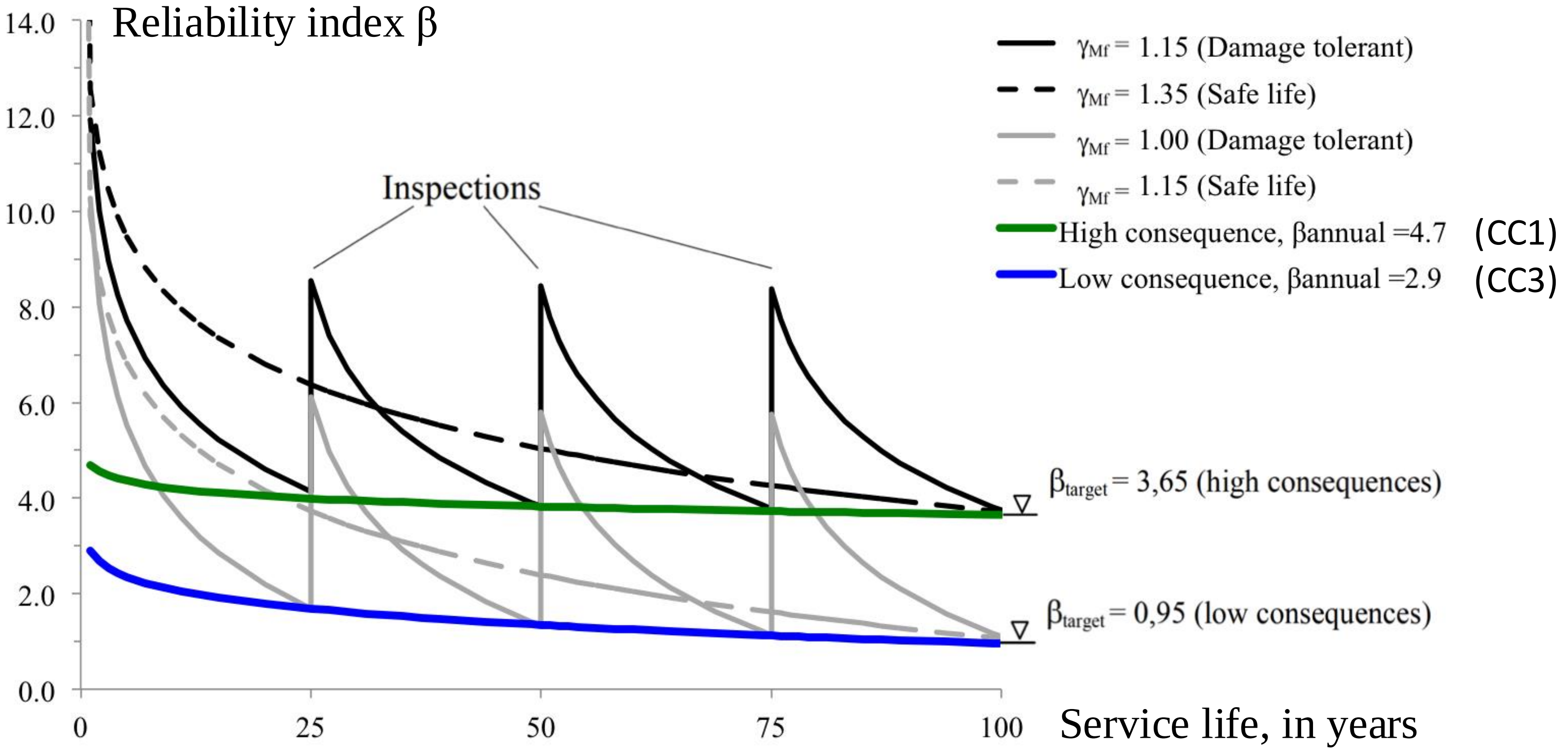
Select appropriate constructional details, materials and stress levels to ensure sufficient reliability level.

For damage tolerance concept : at end of each in-service inspection interval, so that in event of formation of cracks, one or all of following safety mechanisms are ensured:

- low propagation rates and easily detectablecracks prior to failure
- multiple load paths
- crack-arresting constructional details prevent progressive damage.

Examples: details on multi-beams systems, twin-beam details of continuous beam systems, slab reinforcement (because of the large number of rebars), closely spaced hangers, etc.

Evolution of β according to choice of verification method

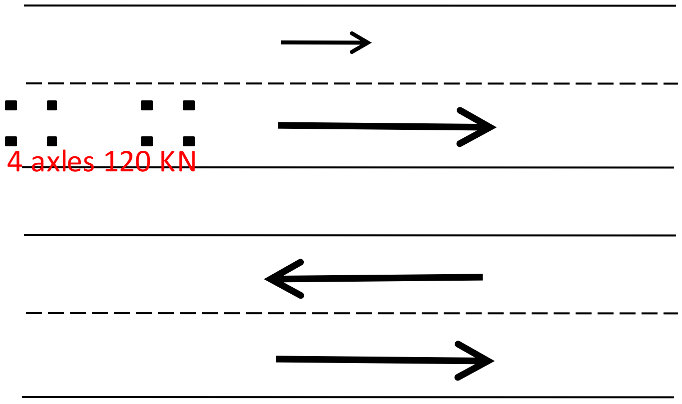


APPENDICES

Eurocodes in relation to fatigue

AT	Title	Fatigue sections
EN 1992-1-1	Concrete, General rules and buildings	§2 bases, §3 steels, §6.8 checks, Appendix C properties
EN 1992-2	Concrete bridges	§2 bases, §6.8 checks, appendix NN lambda factors
EN 1993-1-1	Steel, General rules and buildings	§2 bases, §4 durability
EN 1993-1-9	Fatigue	All
EN 1993-1-10	Choice of steel grades	All
EN 1993-1-11	Calculation of cable structures or tension elements	§2 bases, §7.2 stress limitation, §9 checks and details, appendix A tests
EN 1993-2	Steel bridges	§2 bases, §7.4 web breathing, §9 checks and lambda factors, appendix C constructive rules, details
EN 1993-6	Cranes	§2 bases, §7.4 web breathing, §9 checks and lambda factors
EN 1994-1-1	Mixed, General rules and buildings	§6.8 checks.
EN 1994-2	Mixed bridges	§6.8 checks and factors, Appendix C

Warning: different principles between fatigue load models



Road bridges:

- Traffic, volume, data per year and slow lane
- Single lane load model
- Heavy vehicles mostly on the right (= slow) lane
- Bidirectional case: two slow lanes, probability of crossing? Considered to have a negligible effect
- May be revised (c = calibrated crossing factor in fct of road category):

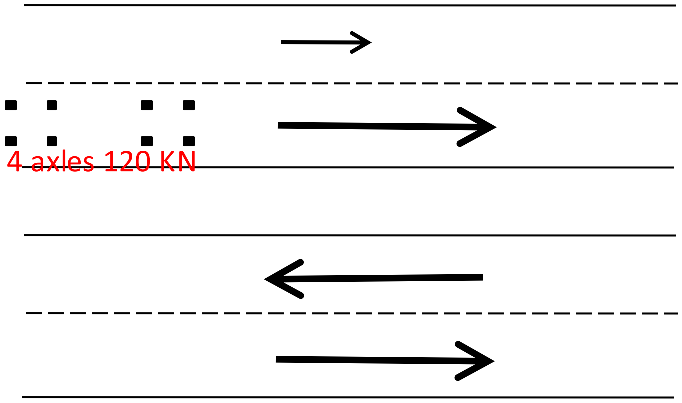
Possible new formulation taking into account crossings (2 lanes):

$$\lambda_4 = \left[(1-c) + \left(\frac{N_2}{N1} - c \right) \left(\frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}} \right)^5 + c \left(1 + \frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}} \right)^5 \right]^{1/5}$$

Warning: different principles between fatigue load models

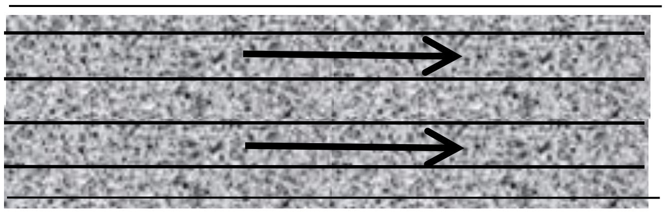
Road bridges:

- Traffic, volume, data per year and slow lane
- Single lane load model
- Heavy vehicles mostly on the right (= slow) lane
- Bidirectional case: two slow lanes, probability of crossing? Considered to have a negligible effect



Rail bridges:

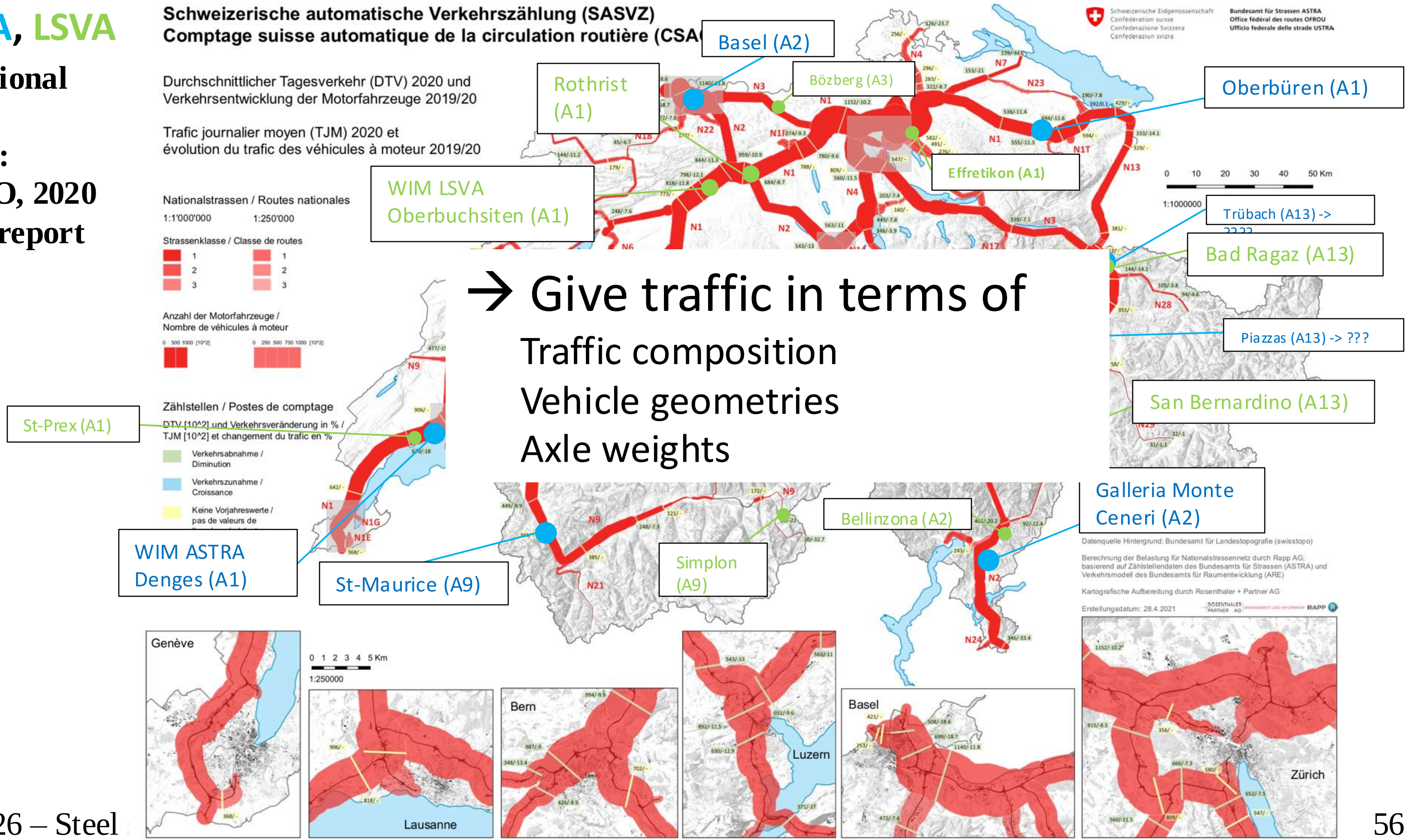
- Traffic, volume, given in millions of tonnes per year and track
- Two track load model
- Trains can often be on the bridge at the same time (close to stations, cannot put timetable constraints to avoid it)
- Unidirectional and bidirectional cases very similar (trains are long), same crossing probability



Damage equivalent factor calibration: based on Swiss traffic, Weigh-in-Motion stations

ASTRA, **LSVA**

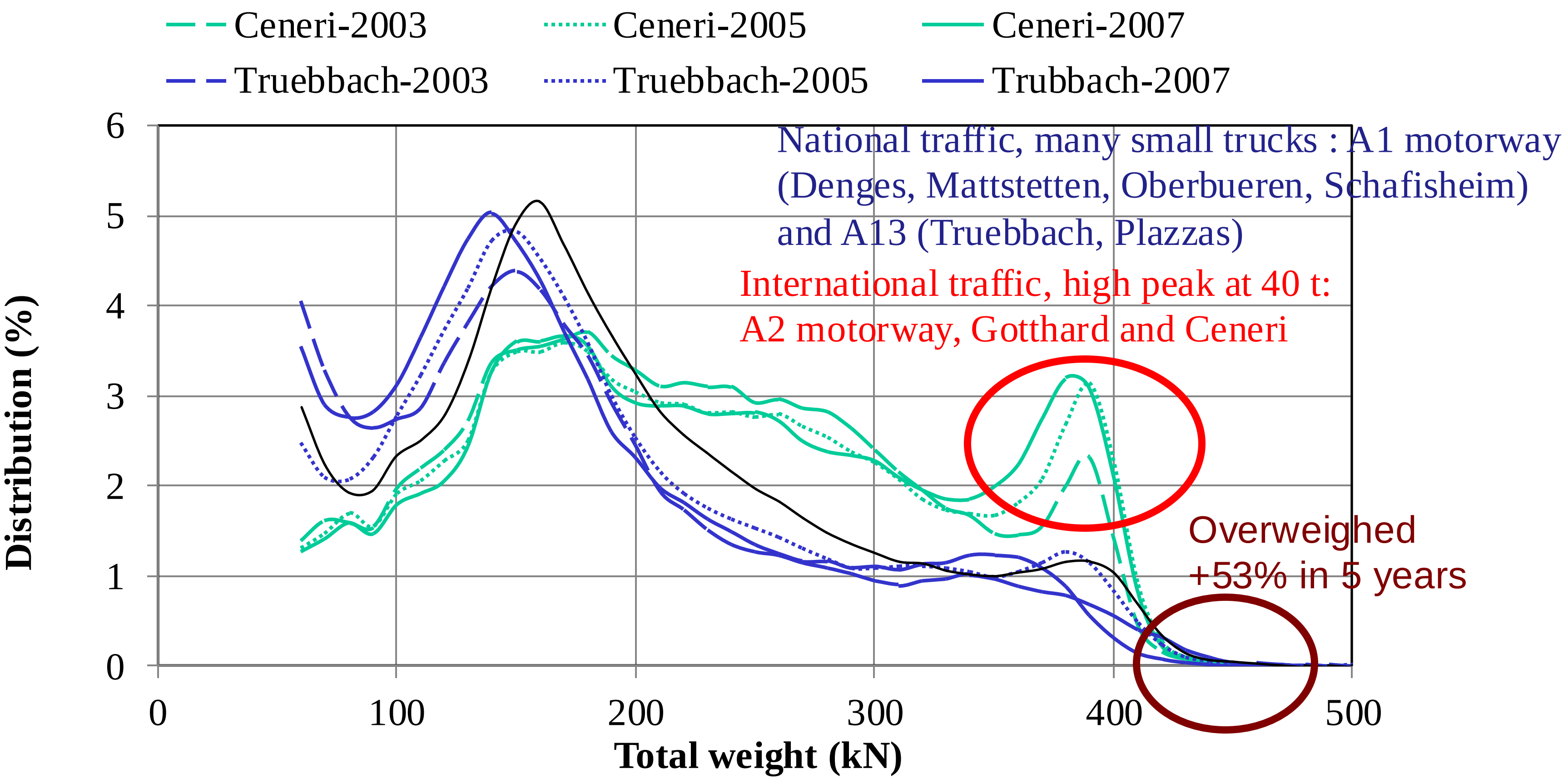
On national roads
Source: FEDRO, 2020 traffic report



→ Give traffic in terms of

- Traffic composition
- Vehicle geometries
- Axle weights

Damage equivalent factor calibration. Parameters: Histograms of total vehicle weights



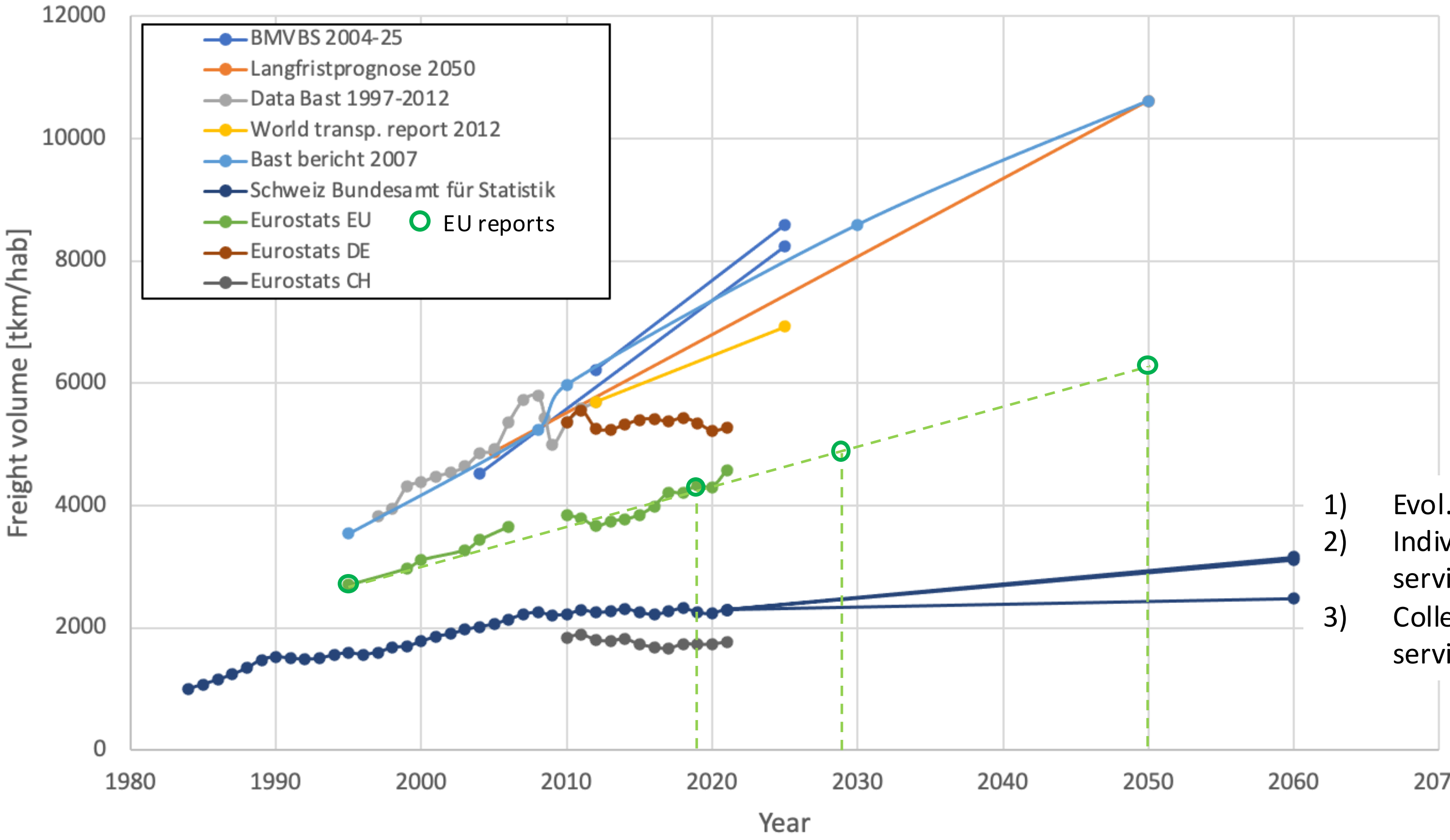
Damage equivalent factor calibration. Parameters: future traffic evolution assumption

- Main basis :
  - Past 30 years
  - ASTRA and Federal Office for Spatial Development (ARE)

    Sources: INFRAS AG (2020), Verkehr der Zukunft 2060: Synthesebericht, SVI 1685 ARE (2016), Transport Perspectives 2040, Doc. ASTRA 82001 (2024)
- In EU various trends and assumptions between countries
- In tonnage, 2010-40: +37%, corr. to 1% annual (both rail and road)
- 3 scenarios for the roads: +8% (mobility revolution) to +37% (evolution)
- AADT growth 1.4% to 1.8%
- ADTT growth 0.35 to 0.4 %

ADTT (Average Daily Truck Traffic)
AADT (Annual Average Daily Traffic)

Damage equivalent factor calibration. Parameters: Normalised freight volume to tons·km/habitant

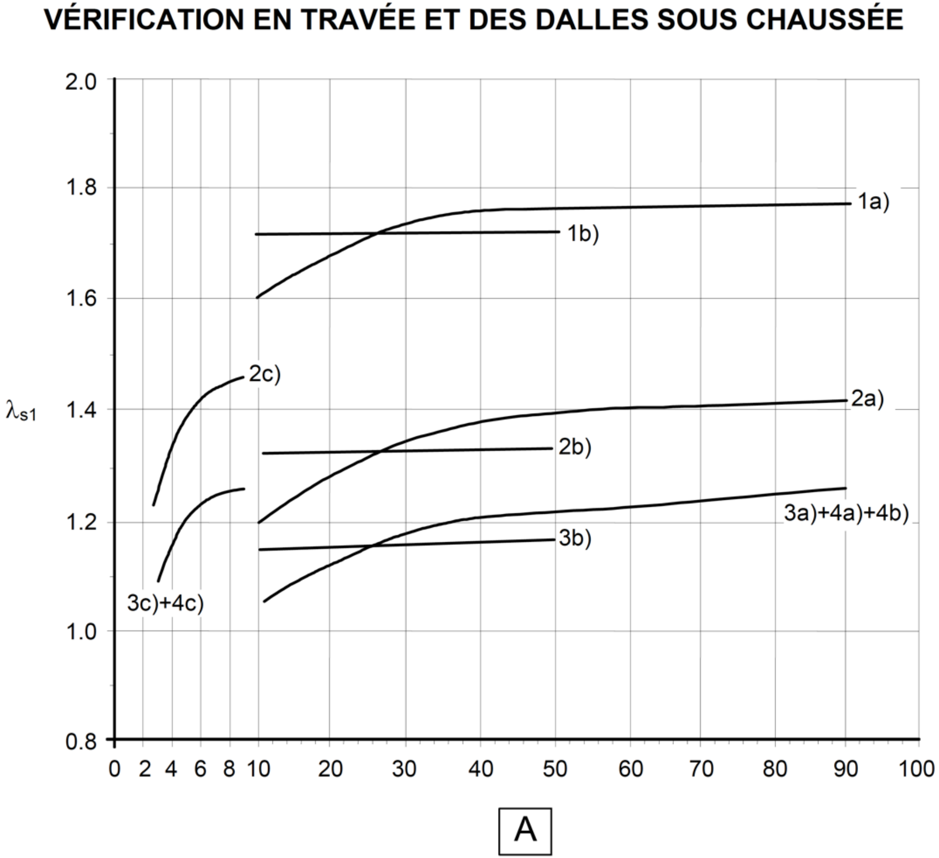


1) Evol. w/o disruption
2) Indiv. mobility services revolution
3) Collective mobility services revolution

Reinforced and prestressed concrete bridges (EN 1992-2) - Reinforcement and steel fatigue curves

SIA263		EN 1992-1-1					
Type d'armature	$\Delta\sigma_{sd,fat}$ [N/mm^2] (k_1 = 4) (k_2 = 7)	EN 1992-1-1 Category	Type d'armature	N*	k_1	k_2	$\Delta\sigma_{Rsk,fat}$ pourr N* cycles [N/mm^2]
Acier d'armature passive		Armatures de béton armé					
barres rectilignes Ø ≤ 20 mm	145	**A1**	Barres droites et barres pliées	10^6	5	9	162.5
barres rectilignes 20 mm < Ø ≤ 40 mm	120						
étriers verticaux Ø ≤ 16 mm façonnés selon le chiffre 5.2.4	135						
– joints longitudinaux soudés	55	**B1**	Barres soudées et treillis soudés	10^7	3	5	58.5
– joints de croisement soudés (treillis, par exemple)							
– liaison mécanique des barres		**B2**	Dispositifs de couplage	10^7	3	5	35
Acier de précontrainte et unités de précontrainte		Armatures de précontrainte					
– unités de précontrainte monotorons sans adhérence	175	**A3**	monotorons dans gaine en matière plastique	10^6	5	9	185
– unités de précontrainte à torons, monocouches,dans des gaines en matière synthétique							
– torons et barres pour la précontrainte par fils adhérents	145	**A2**	Précontrainte par pré-tension	10^6	5	9	185
– unités de précontrainte à fils ou à torons, en plusieurs couches, avec ou sans adhérence, dans des gaines en matière synthétique		**C**	armatures de précontrainte droites ou armatures de précontrainte courbes dans gaines en matière plastique	10^6	5	10	150
unités de précontrainte à fils ou à torons, avec ou sans adhérence, dans des gaines en acier	95	**D**	armatures de précontrainte courbes dans gaines en acier	10^6	5	7	120
ancrages, accouplements	70	**E**	dispositifs de couplage	10^6	5	5	80

Equivalence coef. for reinforced concrete (EN 1992-2)



1) Coupling systems
2) Prestressing rebars curved in steel duct
3) Reinforced concrete steel : pretension (all elements) & post-tension (strands in plastic ducts, straight prestressing rebars)
4) Shear reinforcement

a) Continuous beam
b) Simple supported beam
c) Slab under the road

A d'inf Critical length of the influence line